A Gödelian Ontological Proof with More Plausible Axiological Principles[*]

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ABSTRACT. A drawback of the standard modal ontological proof is that it assumes that it's possible that there is something godlike. Kurt Gödel's ontological proof seeks to establish this possibility with the help of certain axiological principles. The axiological principles he relies on, however, are not very plausible. And the same goes for other Gödelian ontological proofs in the literature. In this paper, I put forward a Gödelian ontological proof that only relies on plausible axiological principles. And I adapt the proof both for constant and varying domains. Nevertheless, the proof still needs the axiom that being godlike is positive in the sense of being a "purely good"-making property.

The standard modal ontological proof for the existence of a godlike being runs as follows: It's possible that something godlike exists. That something godlike exists strictly entails that it's necessary that something godlike exists. Therefore, by standard principles of modal logic, something godlike exists.[1]

A drawback of this proof that it assumes that it's possible that there is something godlike. Kurt Gödel's ontological proof seeks to establish this possibility with the help of certain axiological principles we can show that it is possible that there is something godlike.[2] But the axiological principles he relies on are not very plausible. And the same goes for other Gödelian ontological proofs in the literature. In this paper, I will put forward a Gödelian ontological proof that only relies on plausible axiological principles. And I will adapt the proof for both constant and varying domains.

[*] Forthcoming in *Ergo*.

[†] I would be grateful for any thoughts or comments on this paper, which can be sent to me at *johan.eric.gustafsson@gmail.com*.

[1] Hartshorne 1962, p. 51 and Plantinga 1974a, p. 111; 1974b, p. 214. Nonetheless, Anderson (2015, pp. 286–8), who once defended a Gödelian ontological proof (Anderson 1990), prefers Hartshorne's standard modal ontological proof.

[2] Gödel 1987; 1995a.

Gödel's proof relies on the principle that, for all properties, exactly one of the property and its complement is positive:[3]

(1) $\forall\phi(P(\neg\phi) \equiv \neg P(\phi))$.

Here, $P(\phi)$ states that property ϕ is positive. A *positive* property, is according to Gödel, a property that is 'positive in the moral aesthetic sense'.[4] He adds that positive properties can be interpreted as perfective properties — that is, "purely good"-making properties.[5]

The right-to-left direction of (1) is implausible. There seems to be neutral properties such that neither the property nor its complementary property is positive.[6] The property of being male seems neutral and hence not positive, but the property of not being male also seems neutral and not positive.[7] (If one does not accept that being male seems neutral, one could replace being male with some other neutral property that does not seem to be the negation of a positive property.)

Gödel's proof also relies on the following axiological principle:[8]

(2) $\forall\phi\forall\psi\Big(P(\phi)\ \&\ \Box\forall x(\phi(x) \supset \psi(x)) \supset P(\psi)\Big)$.

Yet this principle is likewise implausible. Let G be the property of being godlike and D be the property of being devil-like. While G seems (as we will assume later on) positive, $G \vee D$ (the property of being godlike or

[3] Gödel 1987, p. 256; 1995a, p. 403. Magari (1988, p. 13), Fitting (2002, pp. 146, 165), and Kovač (2003, p. 572) rely on the same principle. If ϕ and ψ are properties, then $\neg\phi$ is an abbreviation of $\lambda x(\neg(\phi(x)))$, $\phi \vee \psi$ is an abbreviation of $\lambda x(\phi(x) \vee \psi(x))$, and $\phi\ \&\ \psi$ is an abbreviation of $\lambda x(\phi(x)\ \&\ \psi(x))$, where $\lambda x(f(x))$ is a property an individual y has in virtue of being such that $f(y)$. See Carnap 1947, p. 3.

[4] Gödel 1987, p. 257; 1995a, p. 404. Bjørdal (1999, p. 215) defines positivity in terms of godlikeness rather in terms of value — namely, he takes a positive property to be a property such that it's necessary that any godlike being has the property. This does not fit with an axiological interpretation of positivity, since, on Bjørdal's view, tautological properties would be positive (whereas, axiologically, they seem to be neutral rather than positive).

[5] Gödel 1995b, p. 435.

[6] Anderson 1990, p. 295.

[7] Gustafsson 2020, p. 232.

[8] Gödel 1995a, p. 403. Scott (1987, p. 257), Magari (1988, p. 14), Anderson (1990, p. 291), Kovač (2003, p. 572), Maydole (2003, p. 301), Johnson (1999, p. 99; 2004, p. 121), Pruss (2009, p. 347; 2012, pp. 203, 205), and Benzmüller (2020, p. 786; 2022, p. 958) also rely on this principle. And Fitting (2002, p. 165) relies on a varying-domain analogue.

devil-like) does not seem positive.[9] Yet G implies $G \vee D$. So, according to (2), if G is positive, then $G \vee D$ is also positive. But there seems to be no more reason to regard $G \vee D$ as positive than to regard it as negative.[10] (If one does not accept that the magnitude of the negativity of D matches the magnitude of the positivity of G, one could replace G and D with some other pair is positive and negative properties that are alike in magnitude.)

Similarly, tautological properties like $G \vee \neg G$ (the property of being godlike or not godlike) does not seem positive even if one of their disjuncts is positive.[11] But, according to (2), if G is positive, then $G \vee \neg G$ is also positive. It seems that, if (2) holds for the logic of positivity, then the following analogous principle should hold for the logic of negativity, where $N(\phi)$ states that property ϕ is negative:

$$(3) \qquad \forall\phi\forall\psi\Big(N(\phi) \ \& \ \Box\forall x(\phi(x) \supset \psi(x)) \supset N(\psi)\Big).$$

But, if we accept both (2) and (3), we find that $G \vee D$ is both positive and negative, which conflicts with the following principle:[12]

$$(4) \qquad \forall\phi\Big(\neg(P(\phi) \ \& \ N(\phi))\Big).$$

This principle seems plausible given an axiological form of positivity and negativity. It mirrors a standard principle of the logic of value, namely, that nothing is both intrinsically good and intrinsically bad.[13]

Petr Hájek's version of the ontological proof relies on the following axiological principle:[14]

$$(5) \qquad \forall\phi\forall\psi\Big(P(\phi) \ \& \ \Box\forall x(\phi(x) \supset \psi(x)) \supset \neg P(\neg\psi)\Big).$$

This principle states that all positive properties are logically compatible. If any Gödelian ontological proof is to have any success in showing that there is a godlike being that has all positive properties, then those properties need to be logically compatible. It's not clear why we should accept

[9] Hájek 2002, p. 150.

[10] Sobel 2004, p. 122.

[11] Sobel 2004, p. 120; 2006a, pp. 406–7; 2006b, p. 286 and van Inwagen 2007, p. 142.

[12] Gustafsson 2020, p. 233.

[13] Chisholm and Sosa 1966, p. 248

[14] Hájek 2002, p. 156. See also Gödel 1995b, 435 and Cook 2004, p. 106 for similar principles.

this principle, however. The principle has no compelling analogue in the logic of value.

It may seem then that Gödelian ontological proofs need to rely on questionable axiological principles, which does not mirror standard principles in the logic of value. In the following, however, I will show that a Gödelian ontological proof can avoid doing so. We will first consider a constant-domain setting.

1. Constant domain

We adopt the two general formal axiological principles as axioms. The first axiom states that co-entailing properties are alike in positivity:[15]

(C1) $\forall\phi\forall\psi\Big(\Box\forall x\big(\phi(x) \equiv \psi(x)\big) \supset \big(P(\phi) \equiv P(\psi)\big)\Big).$

The second axiom states that contradictory properties aren't positive:[16]

(C2) $\forall\phi\neg P(\phi \mathbin{\&} \neg\phi).$

I have previously shown that these principles are sufficient to derive the first half of Gödel's proof — that is, that, if a property is positive, then it's possible that there exists something that has that property.[17] Here, however, we will show that we can prove the necessary existence of something godlike without assuming any implausible axiological principles.

[15] Gustafsson 2020, p. 235. One may worry that (C1) could fail to hold if we make hyperintensional distinctions among properties (for example, distinguishing being triangular from being trilateral). (See Cresswell 1975, p. 25.) One way to mitigate this worry could be to weaken the principle to just say that, if a first property is logically equivalent to the contradictory property of both having and not having the first property, then those properties are alike in positivity:

(C1*) $\forall\phi\Big(\Box\forall x\big(\phi(x) \equiv \phi \mathbin{\&} \neg\phi(x)\big) \supset \big(P(\phi) \equiv P(\phi \mathbin{\&} \neg\phi)\big)\Big).$

The proof still works if (C1) is replaced by (C1*). Even if one may think that being a triangular differs in positivity from being trilateral due to a hyperintensional distinction, it seems less plausible that the property of being a triangular circle differs in positivity from the property of being both a triangular circle and not a triangular circle. Since both properties involve the same concepts, it seems implausible that a hyperintensional distinction would make a difference for positivity.

[16] Gustafsson (2020, p. 234) instead uses the axiom that the property of being self-different is not positive. But it seems that the reason for believing that the property of being self-different isn't positive is that it is contradictory. So (C2) seems more fundamental.

[17] Gustafsson 2020, pp. 234–6.

We define the property of being godlike as follows:[18]

$$\text{(C3)}\quad G(x) =_{\text{df}} \forall\phi\big(P(\phi) \supset \Box\phi(x)\big).$$

That is, something is godlike if and only if it has all positive properties necessarily. We adopt the following substantial axiological axiom:[19]

$$\text{(C4)}\quad P(G).$$

That is, the property of being godlike is positive. And we seek to prove that it's necessary that there exists something godlike:

$$\text{(C5)}\quad \Box\exists x\, G(x).$$

We can prove the following theorem:

THEOREM 1: Given axioms (C1), (C2), (C3), and (C4), we can derive (C5) in second-order system KB.

For proof, see Appendix A.

Our two general axiological principles, (C1) and (C2), both mirror standard principles of the logic of value.

Axiom (C1) mirrors the principle that logically equivalent states of affairs have the same intrinsic value.[20] There is a straightforward rationale for the principle: If two properties mutually entail each other, any goodness and badness that is entailed by one of them is also entailed by the other. Hence the properties necessarily have the same advantages and disadvantages and so should be alike in positivity.[21]

[18] Anderson (1990, pp. 294-5) has a biconditional instead of a conditional in definiens. Hájek (2002, p. 156) defines being godlike if and only if its necessary properties are those needed to have all positive properties. My definition is weaker in the sense that anything that is godlike by their definitions would also be godlike by mine. Note that only the left-to-right direction of my definition is used in the proofs. So we could replace (C3) by the following:

$$\text{(C3*)}\quad G(x) \supset \Big(\forall\phi\big(P(\phi) \supset \Box\phi(x)\big)\Big).$$

Since (C3*) can be derived from both Anderson's and Hajek's definitions, we could also use their definitions. Anderson's definition, however, has the drawback that — unless tautological properties are positive — godlike beings are impossible.

[19] Scott 1987, p. 257.

[20] Rescher 1966, p. 58 and Åqvist 1968, p. 259.

[21] Gustafsson 2020, p. 235.

Axiom (C2) mirrors the principle that contradictory states of affairs are not intrinsically good.[22] And there is a rationale for the principle too: contradictions entail everything; so, for every good or bad thing that they entail, they also entail the complement. Contradictions are symmetrical in their relation to the good and the bad. Hence they are neither intrinsically good nor positive.[23]

The proof relies on second-order system KB.[24] System K is normal modal logic, that is, propositional logic combined with the necessitation rule and the distribution axiom:[25]

K $\quad \Box(p \supset q) \supset (\Box p \supset \Box q).$

System KB is system K combined with the Brouwerian axiom:[26]

B $\quad p \supset \Box\Diamond p.$

Possibility, here, is defined as the dual of necessity:[27]

$$\Diamond p =_{\mathrm{df}} \neg\Box\neg p.$$

If — in addition to (C5) — we wish to derive

(C6) $\exists x\, G(x),$

we also need the necessity axiom:[28]

T $\quad \Box p \supset p.$

Note that axiom T is not needed to derive (C5).

[22] von Wright 1972, pp. 163–4 and Hansson 2001, p. 119.

[23] Gustafsson 2020, pp. 234–5.

[24] For system KB, see Chellas 1980, p. 131.

[25] For system K, see Chellas 1980, p. 131. For axiom K, see Feys 1950, p. 500 and Chellas 1980, p. 7.

[26] For axiom B, see Lewis and Langford 1932, p. 497 and Chellas 1980, p. 16.

[27] Aristotle *An. pr.* 1.13 32^{a}25; 2009, p. 18, Carnap 1947, p. 186, and Chellas 1980, p. 7.

[28] Carnap 1947, p. 186 and Chellas 1980, p. 6.

2. Varying domain

One worry about Theorem 1 is that it relies on a constant domain.[29] To get around this, we can move to a varying-domain setting with an existence predicate.[30] Let E be a predicate applied to individuals, with $E(x)$ read as 'x concretely exists'. Then we adopt the following definitions:[31]

$$\forall^E x\, \Phi =_{\mathrm{df}} \forall x\big(E(x) \supset \Phi\big).$$
$$\exists^E x\, \Phi =_{\mathrm{df}} \exists x\big(E(x)\ \&\ \Phi\big).$$

We adopt the following axiological principles as axioms, where (V1) is a varying-domain analogue of (C1) and where (V2) is the same as (C2):[32]

(V1) $\forall\phi\forall\psi\Big(\Box\forall^E x\big(\phi(x) \equiv \psi(x)\big) \supset \big(P(\phi) \equiv P(\psi)\big)\Big).$

(V2) $\forall\phi\neg P(\phi\ \&\ \neg\phi).$

We define the property of being godlike as before:[33]

(V3) $G(x) =_{\mathrm{df}} \forall\phi\big(P(\phi) \supset \Box\phi(x)\big).$

We adopt the following substantial axiological axiom:[34]

(V4) $P(G\ \&\ E).$

[29] Another worry is that it seems to allow a parallel argument that there exists something devil-like. Replacing G and P with D and N respectively in (C1)–(C5) seems to yield a similarly plausible argument. If, however, concrete existence is positive and its negation is negative (as suggested in Smullyan 2002, p. 47), then the parallel negative variant of the varying-domain argument below is blocked.

[30] Anderson 1990, pp. 300-1n14.

[31] Fitting and Mendelsohn 1998, p. 106 and Fitting 2002, p. 90.

[32] Gustafsson 2020, p. 236n15. Like before, we could weaken (V1) as follows:

(V1*) $\forall\phi\Big(\Box\forall^E x\big(\phi(x) \equiv \phi\ \&\ \neg\phi(x)\big) \supset \big(P(\phi) \equiv P(\phi\ \&\ \neg\phi)\big)\Big).$

See note 15.

[33] And, like before, we only make use of the left-to-right direction of the definition — that is, we can replace (V3) with the following:

(V3*) $G(x) \supset \Big(\forall\phi\big(P(\phi) \supset \Box\phi(x)\big)\Big).$

See note 18.

[34] Hájek 2002, p. 160.

That is, the property of being godlike and concretely existing is positive. And we seek to prove that it's necessary that there concretely exists something godlike:

(V5) $\Box\exists^E x\, G(x)$.

We can prove the following theorem:

Theorem 2: Given axioms (V1), (V2), (V3), and (V4), we can derive (V5) in second-order system KB.

For proof, see Appendix B.

3. Are these proofs compelling?

Given that (C1), (C2), (V1), and (V2) are plausible axiological principles, are these proofs compelling as arguments for their conclusion? Since (C3) and (V3) are definitions and the required modal system (second-order system KB) is fairly weak, it seems that whether we should accept the argument rests on whether we should accept the remaining axioms, (C4) and (V4). Unlike the other axiological axioms, (C4) and (V4) are not general formal axiological principles — that is, general principles about the formal structure of the positivity or value of properties. They are substantial axiological claims — that is, claims that a specific property is positive.

If we grant that there are positive properties and that they are jointly consistent, it's plausible that G or G & E is positive. The trouble is that it remains unclear that we should grant this.[35]

Moreover, (C4) and (V4) entail, given (C3) or (V3), that the property of necessarily having each positive property is positive. Given that we understand positive properties as "purely good"-making properties, G seems to entail more than just basic good-making properties unless necessarily having a positive property is itself positive.[36] It's unclear why hav-

[35] Anderson and Gettings 1996, p. 171, van Inwagen 2007, p. 144, and Gustafsson 2020, p. 238. Apart from the well known worries about the compatibility of traditional divine properties such as omniscience, omnipotence, and omnibenevolence, there is also the worry that some "purely good"-making properties may only be possessed by objects of different kinds. For example, the property of being a pleasure cannot be possessed by people whereas the property of being benevolent can only be possessed be people. To get around this problem, we can restrict P to properties that apply to a specific kind of object. (I thank Wlodek Rabinowicz for this point.)

[36] Gödel (1995b, p. 435), however, grants that 'the necessity of a positive property is positive'.

ing a possible property in other possible worlds would be good-making in this world.[37]

Having noted these problems, the upshot of this paper is still that, *if* we grant that G or G & E is positive, the conclusion follows. And, unlike earlier Gödelian ontological proofs, we have shown this without relying on implausible axiological principles.

Appendices

A. Proof of Theorem 1

THEOREM 1: Given axioms

(C1) $\forall\phi\forall\psi\Big(\Box\forall x\big(\phi(x) \equiv \psi(x)\big) \supset \big(P(\phi) \equiv P(\psi)\big)\Big)$.

(C2) $\forall\phi\neg P(\phi \;\&\; \neg\phi)$.

(C3) $G(x) =_{\text{df}} \forall\phi\big(P(\phi) \supset \Box\phi(x)\big)$.

(C4) $P(G)$.

we can derive, by second-order system KB,

(C5) $\Box\exists x\, G(x)$.

Proof.[38] Assume, for proof by contradiction,

(6) $\neg\forall\phi\big(P(\phi) \supset \Diamond\exists x\, \phi(x)\big)$. [Assumption]

From (6), we have

(7) $\exists\phi\, \neg\big(P(\phi) \supset \Diamond\exists x\, \phi(x)\big)$. [(6)]

From (7), we have, by existential instantiation,

(8) $\neg\big(P(\phi') \supset \Diamond\exists x\, \phi'(x)\big)$. [(6)]

From (8), we have

(9) $P(\phi')$ [(6)]

[37] I thank Wlodek Rabinowicz for this point.

[38] The proof up to (19) follows Gustafsson 2020, pp. 235–6.

and

(10) $\neg\Diamond\exists x\, \phi'(x)$. [(6)]

From (10), we have, by the definition of possibility,

(11) $\Box\neg\exists x\, \phi'(x)$. [(6)]

From (11), we have, by necessitation and the K axiom,

(12) $\Box\forall x\, \neg\phi'(x)$. [(6)]

From (12), we have, by necessitation and the K axiom,

(13) $\Box\forall x\big(\phi'(x) \supset \phi' \;\&\; \neg\phi'(x)\big)$. [(6)]

We have, by necessitation,

(14) $\Box\forall x\big(\phi' \;\&\; \neg\phi'(x) \supset \phi'(x)\big)$. []

From (13) and (14), we have, by necessitation and the K axiom,

(15) $\Box\forall x\big(\phi'(x) \equiv \phi' \;\&\; \neg\phi'(x)\big)$. [(6)]

From (C1) and (15), we have

(16) $P(\phi') \equiv P(\phi' \;\&\; \neg\phi')$. [(C1), (6)]

From (C2) and (16), we have

(17) $\neg P(\phi')$. [(C1), (C2), (6)]

From (9) and (17), we have

(18) $P(\phi') \;\&\; \neg P(\phi')$. [(C1), (C2), (6)]

Having derived a contradiction from (6), we can conclude, not depending on (6),

(19) $\forall\phi\big(P(\phi) \supset \Diamond\exists x\, \phi(x)\big)$. [(C1), (C2)]

From (C4) and (19), we have

(20) $\Diamond\exists x\, G(x)$. [(C1), (C2)]

Assume, for conditional proof,

(21) $G(a)$. [Assumption]

From (C3) and (21), we have

(22) $\forall\phi\big(P(\phi) \supset \Box\phi(a)\big)$. [(C3), (21)]

From (22), we have

(23) $P(G) \supset \Box G(a)$. [(C3), (21)]

From (C4) and (23), we have

(24) $\Box G(a)$. [(C3), (C4), (21)]

From (24), we have, not depending on (21),

(25) $G(a) \supset \Box G(a)$. [(C3), (C4)]

From (25), we have

(26) $\forall x\big(G(x) \supset \Box G(x)\big)$. [(C3), (C4)]

From (26), we have,

(27) $\exists x\, G(x) \supset \exists x\, \Box G(x)$. [(C3), (C4)]

We have, by necessitation and the K axiom,

(28) $\exists x\, \Box G(x) \supset \Box\exists x\, G(x)$. []

From (27) and (28), we have

(29) $\exists x\, G(x) \supset \Box\exists x\, G(x)$. [(C3), (C4)]

From (29), we have,

(30) $\neg\Box\exists x\, G(x) \supset \neg\exists x\, G(x)$. [(C3), (C4)]

From (30), we have, by necessitation,

(31) $\Box\big(\neg\Box\exists x\, G(x) \supset \neg\exists x\, G(x)\big)$. [(C3), (C4)]

Assume, for proof by contradiction,

(32) $\neg\Box\exists x\, G(x)$. [Assumption]

From (30) and (32), we have

(33) $\neg\exists x\, G(x)$. [(C3), (C4), (32)]

From (33), we have, by the B axiom,

(34) $\Box\Diamond\neg\exists x\, G(x)$. [(C3), (C4), (32)]

From (34), we have, by the definition of possibility,

(35) $\Box\neg\Box\neg\neg\exists x\, G(x)$. [(C3), (C4), (32)]

From (35), we have, by necessitation and the K axiom,

(36) $\Box\neg\Box\exists x\, G(x)$. [(C3), (C4), (32)]

From (31) and (36), we have, by the K axiom,

(37) $\Box\neg\exists x\, G(x)$. [(C3), (C4), (32)]

From (20), we have, by the definition of possibility,

(38) $\neg\Box\neg\exists x\, G(x)$. [(C1), (C2)]

From (37) and (38), we have

(39) $\Box\neg\exists x\, G(x)\ \&\ \neg\Box\neg\exists x\, G(x)$. [(C1), (C2), (C3), (C4), (32)]

Having derived a contradiction from (32), we can conclude, not depending on (32),

(C5) $\Box\exists x\, G(x)$. [(C1), (C2), (C3), (C4)]

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B. Proof of Theorem 2

THEOREM 2: Given axioms

(V1) $\forall\phi\forall\psi\Big(\Box\forall^E x\big(\phi(x) \equiv \psi(x)\big) \supset \big(P(\phi) \equiv P(\psi)\big)\Big)$.

(V2) $\forall\phi\neg P(\phi \,\&\, \neg\phi)$.

(V3) $G(x) =_{\text{df}} \forall\phi\big(P(\phi) \supset \Box\phi(x)\big)$.

(V4) $P(G \,\&\, E)$.

we can derive, by second-order system KB,

(V5) $\Box\exists^E x\, G(x)$.

Proof. Assume, for proof by contradiction,

(40) $\neg\forall\phi\big(P(\phi) \supset \Diamond\exists^E x\, \phi(x)\big)$. [Assumption]

From (40), we have

(41) $\exists\phi\, \neg\big(P(\phi) \supset \Diamond\exists^E x\, \phi(x)\big)$. [(40)]

From (41), we have, by existential instantiation,

(42) $\neg\big(P(\phi') \supset \Diamond\exists^E x\, \phi'(x)\big)$. [(40)]

From (42), we have

(43) $P(\phi')$ [(40)]

and

(44) $\neg\Diamond\exists^E x\, \phi'(x)$. [(40)]

From (44), we have, by the definition of possibility,

(45) $\Box\neg\exists^E x\, \phi'(x)$. [(40)]

From (45), we have, by necessitation and the K axiom,

(46) $\Box\forall^E x\, \neg\phi'(x)$. [(40)]

From (46), we have, by necessitation and the K axiom,

(47) $\Box\forall^E x(\phi'(x) \supset \phi' \ \& \ \neg\phi'(x))$. [(40)]

We have, by necessitation,

(48) $\Box\forall^E x(\phi' \ \& \ \neg\phi'(x) \supset \phi'(x))$. []

From (47) and (48), we have, by necessitation and the K axiom,

(49) $\Box\forall^E x(\phi'(x) \equiv \phi' \ \& \ \neg\phi'(x))$. [(40)]

From (V1) and (49), we have

(50) $P(\phi') \equiv P(\phi' \ \& \ \neg\phi')$. [(V1), (40)]

From (V2) and (50), we have

(51) $\neg P(\phi')$. [(V1), (V2), (40)]

From (43) and (51), we have

(52) $P(\phi') \ \& \ \neg P(\phi')$. [(V1), (V2), (40)]

Having derived a contradiction from (40), we can conclude, not depending on (40),

(53) $\forall\phi(P(\phi) \supset \Diamond\exists^E x \, \phi(x))$. [(V1), (V2)]

From (V4) and (53), we have

(54) $\Diamond\exists^E x \, G(x)$. [(V1), (V2)]

Assume, for conditional proof,

(55) $G(a)$. [Assumption]

From (V3) and (55), we have

(56) $\forall\phi(P(\phi) \supset \Box\phi(a))$. [(V3), (55)]

From (56), we have

(57) $P(G) \supset \Box(G \ \& \ E)(a)$. [(V3), (55)]

From (V4) and (57), we have

(58) $\Box(G \mathbin{\&} E)(a)$. [(V3), (V4), (55)]

From (58), we have, not depending on (55),

(59) $G(a) \supset \Box(G \mathbin{\&} E)(a)$. [(V3), (V4)]

From (59), we have

(60) $\forall^E x\big(G(x) \supset \Box(G \mathbin{\&} E)(x)\big)$. [(V3), (V4)]

From (60), we have, by necessitation and the K axiom,

(61) $\exists^E x\, G(x) \supset \Box\exists^E x\, G(x)$. [(V3), (V4)]

From (61), we have

(62) $\neg\Box\exists^E x\, G(x) \supset \neg\exists^E x\, G(x)$. [(V3), (V4)]

From (62), we have, by necessitation,

(63) $\Box\big(\neg\Box\exists^E x\, G(x) \supset \neg\exists^E x\, G(x)\big)$. [(V3), (V4)]

Assume, for proof by contradiction,

(64) $\neg\Box\exists^E x\, G(x)$. [Assumption]

From (62) and (64), we have

(65) $\neg\exists^E x\, G(x)$. [(V3), (V4), (64)]

From (65), we have, by the B axiom,

(66) $\Box\Diamond\neg\exists^E x\, G(x)$. [(V3), (V4), (64)]

From (66), we have, by the definition of possibility,

(67) $\Box\neg\Box\neg\neg\exists^E x\, G(x)$. [(V3), (V4), (64)]

From (67), we have, by necessitation and the K axiom,

(68) $\Box\neg\Box\exists^E x\, G(x)$. [(V3), (V4), (64)]

From (63) and (68), we have, by the K axiom,

(69) $\Box\neg\exists^E x\, G(x)$. [(V3), (V4), (64)]

From (54), we have, by the definition of possibility,

(70) $\neg\Box\neg\exists^E x\, G(x)$. [(V1), (V2)]

From (69) and (70), we have

(71) $\Box\neg\exists^E x\, G(x)\ \&\ \neg\Box\neg\exists^E x\, G(x)$. [(V1), (V2), (V3), (V4), (64)]

Having derived a contradiction from (64), we can conclude, not depending on (64),

(V5) $\Box\exists^E x\, G(x)$. [(V1), (V2), (V3), (V4)]

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