Decisions under Ignorance and the Individuation of States of Nature

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ABSTRACT: How do you make decisions under ignorance, that is, when you are ignorant of any probabilities for different states of nature? According to the Laplace Rule, you should assign an equal probability to each state of nature (that is, use the Principle of Insufficient Reason), and then maximize expected utility. The most influential objection to this rule is that it is sensitive to the individuation of states of nature. This is problematic since the individuation of states seems arbitrary. In this paper, I show that this objection proves too much. I show that all plausible rules for decisions under ignorance will be sensitive to the individuation of states of nature.

How do you make decisions under ignorance? In other words, how do you make decisions when you are ignorant of any probabilities for different states of nature? A classic answer is

> *The Laplace Rule* Assign an equal probability to each state of nature (that is, use the Principle of Insufficient Reason), and then maximize expected utility.[1]

But the Laplace rule suffers from a well-known problem: It is sensitive to how states of nature are individuated. The problem is that the individuation of states of nature seem arbitrary, so, if the prescriptions of the Laplace Rule depends on that individuation, its prescriptions will be arbitrary too.[2] More formally, the Laplace Rule violates:

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[1] Milnor 1954, pp. 49–50. See also Keynes 1921, pp. 41–42.

[2] See the examples in Keynes 1921, pp. 42–44.

State-Individuation Invariance Whether option x is at least as preferred as option y does not depend on whether a state of nature is split into two duplicate states.[3]

Consider, for instance, the following situation, where S_1 and S_2 are the possible states of nature and we have a choice between options o_1 and o_2:

Situation One

	S_1	S_2
o_1	8	2
o_2	1	7

Applying the Principle of Insufficient Reason, we assign an equal probability to each state of nature: $P(S_1) = P(S_2) = 1/2$. Accordingly, the Laplace Rule entails that o_1 ought to be chosen, since the expected utility of o_1 is $8 * 1/2 + 2 * 1/2 = 5$ and the expected utility of o_2 is $1 * 1/2 + 7 * 1/2 = 4$. But suppose we individuate states of nature more finely so that we split state S_2 into two separate states, S_2' and S_2'':

Situation Two

	S_1	S_2'	S_2''
o_1	8	2	2
o_2	1	7	7

This time, when we assign an equal probability to each state of nature according to the Principle of Insufficient Reason, we get $P(S_1) = P(S_2') = P(S_2'') = 1/3$. And then the Laplace Rule entails that o_2 ought to be chosen, since the expected utility of o_1 is $8 * 1/3 + 2 * 1/3 + 2 * 1/3 = 4$ and the expected utility of o_2 is $1 * 1/3 + 7 * 1/3 + 7 * 1/3 = 5$. Hence splitting S_2 into two duplicate states of natures changes the recommendation of the Laplace Rule.

Ken Binmore takes this individuation sensitivity to be a fatal problem for the Laplace Rule. If we are completely ignorant about the states of the world, he asks, why should we accept a principle that tells us that a duplicated state corresponds to two different but equi-probable states of the world?[4] State-Individuation Invariance is, according to Binmore, one

[3] Milnor (1954, p. 52), Arrow and Hurwicz 1977, p. 466, and Binmore (2009, p. 158) Milnor and Binmore call this principle 'Column Duplication'. Barbarà and Jackson (1988, p. 37) call it 'Independence of Duplicate States'.

[4] Binmore 2009, p. 159. See also Rawls 1974, pp. 649–650.

of the indispensable axioms for decisions under ignorance, along with the following principles:[5]

> *Transitivity* If option x is at least as preferred as option y and y is at least as preferred as option z, then x is at least as preferred as z.[6]
>
> *State Anonymity* If the outcome of option x is just like the outcome of option y except for a permutation of the states of nature (where no state is subjectively more probable than the others), then x is equally preferred as y.[7]
>
> *The Weak Principle of Statewise Dominance* If the outcome of option x is preferred to the outcome of option y in every state of nature, then x is preferred to y.[8]
>
> *Expansion Consistency* Whether option x is at least as preferred as option y does not change if another option is added to the situation.[9]

In this paper, I shall argue that, giving a very compelling strengthening of the Weak Principle of Statewise Dominance, these indispensable axioms actually rule out State-Individuation Invariance. So it's not only the Laplace Rule that sensitive to the individuation of states of nature: Any plausible principle for how to make decisions under ignorance will be so too. Hence the main objection to the Laplace Rule proves too much, since it also rules out all plausible alternatives to the Laplace Rule.

Instead of the Weak Principle of Statewise Dominance, we shall rely on

[5] Binmore 2009, p. 158.

[6] Arrow 1951, p. 13 and Jensen 1967, p. 171. Milnor (1954, p. 51) and Binmore (2009, p. 157) rely on the stronger 'Ordering' principle, which also implies that 'at least as preferred as' is complete—that is either option x is at least as preferred as option y or y is at least as preferred as x.

[7] Milnor (1954, p. 51), Arrow and Hurwicz 1977, p. 466, and Binmore (2009, p. 157) Milnor and Binmore call this principle 'Symmetry'.

[8] Milnor (1954, p. 51), Arrow and Hurwicz 1977, p. 466, and Binmore (2009, p. 157) Milnor and Binmore call this principle 'Strong Domination'.

[9] Milnor (1954, p. 51) and Binmore (2009, p. 157) call this principle 'Row Adjunction'. See also Nash 1950, p. 159, Radner and Marschak 1954, p. 63, and Sen 1969, p. 384.

The Strong Principle of Statewise Dominance If the outcome of option x is at least as preferred as the outcome of option y in every state of nature and the outcome of x is preferred to the outcome of y in one state of nature, then x is preferred to y.[10]

This principle is stronger than the Weak Principle of Statewise Dominance, but both principles are supported by the same kind of thought: An option that is at least preferred as another option in all states of nature and strictly preferred in at least one state to the other option, has no disadvantage to the other option but at least one advantage; hence it should be preferred.

Consider the following situation, where the numbers represent the utilities of the outcomes of options o_1 and o_2 in the two states of nature S_1 and S_2:

Situation Three

	S_1	S_2
o_1	2	1
o_2	1	2

By State Anonymity, we have

(1) In Situation Three, o_1 is equally preferred as o_2.

Consider the following situation, which is just like Situation Three except that we have split S_2 into two separate states of nature, S_2' and S_2'':

Situation Four

	S_1	S_2'	S_2''
o_1	2	1	1
o_2	1	2	2

From (1), we have, by State-Individuation Invariance,

(2) In Situation Four, o_1 is equally preferred as o_2.

Next, consider the following situation, which is just like Situation Four except that we have added another option:

[10] This is a widely accepted principle. See, for example, Savage 1951, p. 58, Milnor 1954, p. 55, Luce and Raiffa 1957, p. 287, Nozick 1969, p. 118, and Barbarà and Jackson 1988, p. 37.

Situation Five

	S_1	S_2'	S_2''
o_1	2	1	1
o_2	1	2	2
o_3	2	2	1

From (2), we have, by Expansion Consistency,

(3) In Situation Five, o_1 is equally preferred as o_2.

Now, consider the following situation, which is just like Situation Five except that we have removed o_1:

Situation Six

	S_1	S_2'	S_2''
o_2	1	2	2
o_3	2	2	1

We have, by State Anonymity,

(4) In Situation Six, o_2 is equally preferred as o_3.

From (4), we have, by Expansion Consistency,

(5) In Situation Five, o_2 is equally preferred as o_3.

From (3) and (5), we have, by Transitivity,

(6) In Situation Five, o_1 is equally preferred as o_3.

But (6) violates the Strong Principle of Statewise Dominance. Hence we have that no rule could satisfy State-Individuation Invariance, Transitivity, State Anonymity, Expansion Consistency, and the Strong Principle of Statewise Dominance. So, even if we reject the Laplace Rule, it is still implausible to reject State-Individuation Invariance, since Transitivity, State Anonymity, Expansion Consistency, and the Strong Principle of Statewise Dominance are all plausible.[11]

[11] The Maximin Rule (Wald 1950, p. 18) violates the Strong Principle of Statewise Dominance. The Leximin Rule (Sen 1970, p. 138n12; 2017, p. 195n12 and Barbarà and Jackson 1988, p. 36) violates State-Individuation Invariance. The Protective Criterion (Barbarà and Jackson 1988, p. 37) violates Transitivity. The Hurwicz Rule (Milnor 1954, p. 50) violates the Strong Principle of Statewise Dominance. The Minimax-Regret Rule (Savage 1951, p. 59) violates Expansion Consistency and the Strong Principle of Statewise Dominance.

This result, of course, does not dissolve the problem of sensitivity to the individuation of states of nature. It just shows that it's a problem for all plausible rules for decisions under uncertainty. The problem is that, if the rule's prescriptions depend on the individuation of states of nature and this individuation is arbitrary, then the rule's prescriptions will be arbitrary. This problem would be solved if we found a non-arbitrary principle of individuation for states of nature. Could we find one? A compelling idea is to adopt

> *The Principle of Individuation by Justifiers* Outcomes should be distinguished as different if and only if they differ in a way that makes it rational to have a preference between them.[12]

Then we could adopt

> *The Principle of Individuation by Outcomes* States of nature should be distinguished as different if and only if some logically possible option has different outcomes in the states.

We need logical possibility here rather than mere feasibility, because if we claimed that *states of nature should be distinguished as different if and only if some feasible option has different outcomes in the states* then the Laplace Rule would violate Expansion Consistency. It would do so because the individuation of states of nature would depend on what options are feasible in the situation. A problem is that this proposal would probably explode the number of states of nature, because, for any arbitrary split of a state of nature, there would probably always be some logically possible act that has different outcomes in the new states.

The point of this paper, however, isn't to solve the problem of how to individuate states of nature. The point is merely that this problem is a problem for all plausible proposals for how to make decisions under ignorance—and not just the Laplace Rule.

[12] Broome 1991, p. 103. One potential worry is whether the Principle of Individuation by Justifiers respects the transitivity of identity. Consider Luce's (1956, p. 179) example with three coffee cups (which is a variation of an example in Armstrong 1939, p. 457n1): Cup *A* has no sugar, cup *B* has one grain of sugar, and cup *C* has two grains of sugar. You can't tell the difference between *A* and *B*. Nor can you tell the difference between *B* and *C*. But you can tell the difference between *A* and *C*. When you can't tell the difference between two options, it's arguably irrational to have a preference between them. Yet it seems rational to have a preference between *A* and *C*. So we get that $A = B = C \neq A$, which violates the transitivity of identity. This objection is blocked, however, if rational indifference is transitive; see Gustafsson 2010.

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