Decisions under Ignorance and the Individuation of States of Nature

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Abstract: How do you make decisions under ignorance, that is, when you are ignorant of any probabilities for different states of nature? According to the Laplace Rule, you should assign an equal probability to each state of nature (that is, use the Principle of Insufficient Reason), and then maximize expected utility. The most influential objection to this rule is that it is sensitive to the individuation of states of nature. This is problematic since the individuation of states seems arbitrary. In this paper, I show that this objection proves too much. I show that all plausible rules for decisions under ignorance will be sensitive to the individuation of states of nature.

How do you make decisions under ignorance? In other words, how do you make decisions when you are ignorant of any probabilities for different states of nature? A classic answer is

The Laplace Rule Assign an equal probability to each state of nature (that is, use the Principle of Insufficient Reason), and then maximize expected utility.¹

But the Laplace rule suffers from a well-known problem: It is sensitive to how states of nature are individuated. The problem is that the individuation of states of nature seem arbitrary, so, if the prescriptions of the Laplace Rule depends on that individuation, its prescriptions will be arbitrary too.² More formally, the Laplace Rule violates:

* I would be grateful for any thoughts or comments on this paper, which can be sent to me at johan.eric.gustafsson@gmail.com.

¹ Milnor 1954, pp. 49–50. See also Keynes 1921, pp. 41–42.
² See the examples in Keynes 1921, pp. 42–44.
State-Individuation Invariance  Whether option $x$ is at least as preferred as option $y$ does not depend on whether a state of nature is split into two duplicate states.

Consider, for instance, the following situation, where $S_1$ and $S_2$ are the possible states of nature and we have a choice between options $o_1$ and $o_2$:

**Situation One**

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_1$</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>$o_2$</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

Applying the Principle of Insufficient Reason, we assign an equal probability to each state of nature: $P(S_1) = P(S_2) = 1/2$. Accordingly, the Laplace Rule entails that $o_1$ ought to be chosen, since the expected utility of $o_1$ is $8 \times 1/2 + 2 \times 1/2 = 5$ and the expected utility of $o_2$ is $1 \times 1/2 + 7 \times 1/2 = 4$. But suppose we individuate states of nature more finely so that we split state $S_2$ into two separate states, $S'_2$ and $S''_2$:

**Situation Two**

<table>
<thead>
<tr>
<th></th>
<th>$S_1$</th>
<th>$S'_2$</th>
<th>$S''_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_1$</td>
<td>8</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$o_2$</td>
<td>1</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

This time, when we assign an equal probability to each state of nature according to the Principle of Insufficient Reason, we get $P(S_1) = P(S'_2) = P(S''_2) = 1/3$. And then the Laplace Rule entails that $o_2$ ought to be chosen, since the expected utility of $o_1$ is $8 \times 1/3 + 2 \times 1/3 + 2 \times 1/3 = 4$ and the expected utility of $o_2$ is $1 \times 1/3 + 7 \times 1/3 + 7 \times 1/3 = 5$. Hence splitting $S_2$ into two duplicate states of natures changes the recommendation of the Laplace Rule.

Ken Binmore takes this individuation sensitivity to be a fatal problem for the Laplace Rule. If we are completely ignorant about the states of the world, he asks, why should we accept a principle that tells us that a duplicated state corresponds to two different but equi-probable states of the world?


of the indispensable axioms for decisions under ignorance, along with the following principles:

Transitivity  If option \( x \) is at least as preferred as option \( y \) and \( y \) is at least as preferred as option \( z \), then \( x \) is at least as preferred as \( z \).\(^6\)

State Anonymity  If the outcome of option \( x \) is just like the outcome of option \( y \) except for a permutation of the states of nature (where no state is subjectively more probable than the others), then \( x \) is equally preferred as \( y \).\(^7\)

The Weak Principle of Statewise Dominance  If the outcome of option \( x \) is preferred to the outcome of option \( y \) in every state of nature, then \( x \) is preferred to \( y \).\(^8\)

Expansion Consistency  Whether option \( x \) is at least as preferred as option \( y \) does not change if another option is added to the situation.\(^9\)

In this paper, I shall argue that, giving a very compelling strengthening of the Weak Principle of Statewise Dominance, these indispensable axioms actually rule out State-Individuation Invariance. So it’s not only the Laplace Rule that sensitive to the individuation of states of nature: Any plausible principle for how to make decisions under ignorance will be so too. Hence the main objection to the Laplace Rule proves too much, since it also rules out all plausible alternatives to the Laplace Rule.

Instead of the Weak Principle of Statewise Dominance, we shall rely on

\(^5\) Binmore 2009, p. 158.
\(^6\) Arrow 1951, p. 13 and Jensen 1967, p. 171. Milnor (1954, p. 51) and Binmore (2009, p. 157) rely on the stronger ‘Ordering’ principle, which also implies that ‘at least as preferred as’ is complete—that is either option \( x \) is at least as preferred as option \( y \) or \( y \) is at least as preferred as \( x \).


**The Strong Principle of Statewise Dominance**  If the outcome of option \( x \) is at least as preferred as the outcome of option \( y \) in every state of nature and the outcome of \( x \) is preferred to the outcome of \( y \) in one state of nature, then \( x \) is preferred to \( y \).\(^{10}\)

This principle is stronger than the Weak Principle of Statewise Dominance, but both principles are supported by the same kind of thought: An option that is at least preferred as another option in all states of nature and strictly preferred in at least one state to the other option, has no disadvantage to the other option but at least one advantage; hence it should be preferred.

Consider the following situation, where the numbers represent the utilities of the outcomes of options \( o_1 \) and \( o_2 \) in the two states of nature \( S_1 \) and \( S_2 \):

\[
\begin{array}{c|cc}
\text{Situation Three} & S_1 & S_2 \\
\hline
o_1 & 2 & 1 \\
o_2 & 1 & 2 \\
\end{array}
\]

By State Anonymity, we have

\[(1) \quad \text{In Situation Three, } o_1 \text{ is equally preferred as } o_2.\]

Consider the following situation, which is just like Situation Three except that we have split \( S_2 \) into two separate states of nature, \( S'_2 \) and \( S''_2 \):

\[
\begin{array}{c|ccc}
\text{Situation Four} & S_1 & S'_2 & S''_2 \\
\hline
o_1 & 2 & 1 & 1 \\
o_2 & 1 & 2 & 2 \\
\end{array}
\]

From (1), we have, by State-Individuation Invariance,

\[(2) \quad \text{In Situation Four, } o_1 \text{ is equally preferred as } o_2.\]

Next, consider the following situation, which is just like Situation Four except that we have added another option:

\(^{10}\) This is a widely accepted principle. See, for example, Savage 1951, p. 58, Milnor 1954, p. 55, Luce and Raiffa 1957, p. 287, Nozick 1969, p. 118, and Barbarà and Jackson 1988, p. 37.
From (2), we have, by Expansion Consistency,

(3) In Situation Five, $o_1$ is equally preferred as $o_2$.

Now, consider the following situation, which is just like Situation Five except that we have removed $o_1$:

**Situation Six**

<table>
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<tr>
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<th>$S_1$</th>
<th>$S'_2$</th>
<th>$S''_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$o_2$</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$o_3$</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

We have, by State Anonymity,

(4) In Situation Six, $o_2$ is equally preferred as $o_3$.

From (4), we have, by Expansion Consistency,

(5) In Situation Five, $o_2$ is equally preferred as $o_3$.

From (3) and (5), we have, by Transitivity,

(6) In Situation Five, $o_1$ is equally preferred as $o_3$.

But (6) violates the Strong Principle of Statewise Dominance. Hence we have that no rule could satisfy State-Individuation Invariance, Transitivity, State Anonymity, Expansion Consistency, and the Strong Principle of Statewise Dominance. So, even if we reject the Laplace Rule, it is still implausible to reject State-Individuation Invariance, since Transitivity, State Anonymity, Expansion Consistency, and the Strong Principle of Statewise Dominance are all plausible.\(^{11}\)

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This result, of course, does not dissolve the problem of sensitivity to the individuation of states of nature. It just shows that it’s a problem for all plausible rules for decisions under uncertainty. The problem is that, if the rule’s prescriptions depend on the individuation of states of nature and this individuation is arbitrary, then the rule’s prescriptions will be arbitrary. This problem would be solved if we found a non-arbitrary principle of individuation for states of nature. Could we find one? A compelling idea is to adopt

*The Principle of Individuation by Justifiers*  Outcomes should be distinguished as different if and only if they differ in a way that makes it rational to have a preference between them.¹²

Then we could adopt

*The Principle of Individuation by Outcomes*  States of nature should be distinguished as different if and only if some logically possible option has different outcomes in the states.

We need logical possibility here rather than mere feasibility, because if we claimed that *states of nature should be distinguished as different if and only if some feasible option has different outcomes in the states* then the Laplace Rule would violate Expansion Consistency. It would do so because the individuation of states of nature would depend on what options are feasible in the situation. A problem is that this proposal would probably explode the number of states of nature, because, for any arbitrary split of a state of nature, there would probably always be some logically possible act that has different outcomes in the new states.

The point of this paper, however, isn’t to solve the problem of how to individuate states of nature. The point is merely that this problem is a problem for all plausible proposals for how to make decisions under ignorance—and not just the Laplace Rule.

¹² Broome 1991, p. 103. One potential worry is whether the Principle of Individuation by Justifiers respects the transitivity of identity. Consider Luce’s (1956, p. 179) example with three coffee cups (which is a variation of an example in Armstrong 1939, p. 457n1): Cup A has no sugar, cup B has one grain of sugar, and cup C has two grains of sugar. You can’t tell the difference between A and B. Nor can you tell the difference between B and C. But you can tell the difference between A and C. When you can’t tell the difference between two options, it’s arguably irrational to have a preference between them. Yet it seems rational to have a preference between A and C. So we get that \( A = B = C \neq A \), which violates the transitivity of identity. This objection is blocked, however, if rational indifference is transitive; see Gustafsson 2010.
I wish to thank John Bone and Richard Pettigrew for valuable comments.

References


Radner, Roy and Jacob Marschak (1954) ‘Note on Some Proposed Decision


