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Money Pumps, Incompleteness, and Indeterminacy

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In an alleged counter-example to the completeness of rational preferences, a career as a clarinettist is compared with a career in law. It seems reasonable to neither want to judge that the law career is at least as preferred as the clarinet career nor want to judge that the clarinet career is at least as preferred as the law career. The two standard interpretations of examples of this kind are, first, that the examples show that preferences are rationally permitted to be incomplete and, second, that the examples show that preferences are rationally permitted to be indeterminate. In this paper, I shall argue that the difference between these interpretations is crucial for the money-pump argument for transitivity, which is the standard argument that rational preferences are transitive. I shall argue that the money-pump argument for transitivity fails if preferences are rationally permitted to be incomplete but that it works if preferences are rationally permitted to be indeterminate and rationally required to be complete.

Two closely related claims about preferences are, first, that they are rationally required to be complete and, second, that they are rationally required to be determinate. Preferences are complete if and only if, for all x and y, either x is at least as preferred as y or y is at least as preferred as x. And preferences are determinate if and only if, for all x and y, either it is determinate that x is at least as preferred as y or it is determinate that x is not at least as preferred as y. These two claims are related in that alleged counter-examples to the first could alternatively be interpreted as merely counter-examples to the second, and vice versa. Consider for instance an alleged counter-example to the completeness of rational preferences where a career as a clarinettist is compared with a career in law.[1] It seems reasonable to neither want to judge that the law career is at least as preferred as the clarinet career nor want to judge that the clarinet career is at least as preferred as the law career. In addition, there are many similar examples of seemingly incomplete rational preferences.[2] Hence

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[1] Raz (1985–6, p. 126).

[2] See, for example, de Sousa (1974, pp. 544–545), Sinnott-Armstrong (1985, p. 327), and Chang (2002, p. 669).

one might conclude that rational preferences need not be complete. Yet there is an alternative interpretation of these examples, on which it is still rationally required that preferences are complete. On this alternative interpretation, these alleged counter-examples to the completeness of rational preferences are merely counter-examples to the determinateness of rational preferences.[3] Perhaps it is rationally permitted that preferences are indeterminate, that is, for **[p. 61]** some x and y, it is indeterminate or vague whether x is at least as preferred as y. The unwillingness to judge which of the clarinet career and the law career one prefers could be due to it being indeterminate whether the first career is at least as preferred as the second and indeterminate whether the second career is at least as preferred as the first, even though it is determinate that one of them is at least as preferred as the other. Thus rational preferences could be indeterminate yet complete, that is, it could be determinate for all x and y that either x is at least as preferred as y or y is at least as preferred as x; but for some x and y it is indeterminate whether x is at least as preferred as y. Hence one might claim that these examples do not show that it is rationally permitted to have incomplete preferences, since these examples might just be examples where it is rationally permitted to have indeterminate but still complete preferences.

While (i) that preferences are rationally permitted to be incomplete and (ii) that preferences are rationally required to be complete yet rationally permitted to be indeterminate are distinct possibilities, one might wonder whether the difference between (i) and (ii) is of any importance.[4] In this paper, I shall argue that this difference is crucial for the money-pump argument for transitivity, which is the standard argument that rational preferences are transitive.[5] Preferences are transitive if and only if, for all x, y, and z, if x is at least as preferred as y and y is at least as preferred as z, then x is at least as preferred as z. I shall argue that the money-pump argument for transitivity fails if preferences are rationally permitted to be incomplete but that it works if preferences are rationally permitted to be indeterminate yet required to be complete, or at least, if

[3] See, for example, Griffin (1986, p. 96), Broome (1997), Rabinowicz (2009, p. 74), and Gustafsson (2013a).

[4] Broome (2004, p. 185), for example, claims that the difference between vagueness and incommensurability in value relations does not make much difference in practice.

[5] The money-pump argument was first presented in Davidson et al. (1955, p. 146). The target of my arguments in this paper is primarily the money-pump argument for transitivity. There is also a version of the money-pump argument that only aims to show that rational preferences are acyclic. Unlike the money-pump argument for transitivity, the money-pump argument for acyclicity is not open to problems with incompleteness. Nevertheless, the problems I shall present with indeterminacy are equally problematic for the argument for acyclicity as for the argument for transitivity.

the argument fails, it is not because of any problems with indeterminacy.[6]

Money-pump arguments aim to show that agents who violate a certain alleged requirement are in some possible situations forced to violate a dominance principle. The idea is that, if some preferences are rationally permissible, then there should not be any possible situation where having those preferences forces one to violate a requirement of rationality, or at least not if the situation just involves a finite number of alternatives.[7] The following are three standard dominance principles: **[p. 62]**

> *The synchronic dominance principle*
> It is rationally required that one does not choose an alternative to which another alternative is preferred.
>
> *The diachronic dominance principle*
> It is rationally required that, if one can foresee which sequences of choices one can make, one does not make a sequence of choices to which an alternative sequence of choices is preferred.
>
> *The monetary dominance principle*
> It is rationally required that, if one can foresee which sequences of choices one can make, one does not make a sequence of choices that yields the same outcome as an alternative sequence of choices but with less money.

One might object that the term '*money*-pump argument' should be reserved for arguments that show that agents who violates a certain alleged requirement are exploitable in the sense that they are in some possible situation forced to violate the monetary dominance principle.[8] Yet, for the purposes of a discussion of rationality, I do not see much value in making such a reservation, because there does not seem to be any special relationship between rationality and money. If one prefers to be poorer

[6] One might object that these alleged counter-examples need to make use of transitivity to rule out indifference; see the small-improvement argument in de Sousa (1974, pp. 544–545), Broome (1978, p. 330), and Raz (1985–6, pp. 120–121). The reason we can rule out that one is merely indifferent between, for example, the career as a clarinettist and the career in law is that a small improvement of one of them does not make it preferred to the other. If one is rational and rational preferences are transitive, we have that one is not indifferent between the careers. Hence it might seem that these counter-examples presuppose transitivity. But the transitivity premise in these counter-examples can be weakened, see Carlson (2011).

[7] If we allow situations with an infinite number of alternatives, we have some cases where every alternative is ruled out by dominance even given what seems to be fully rational preferences. See, for example, Nozick (1963, p. 89) and Arntzenius et al. (2004). The correct diagnosis of what, if anything, it is rational to choose in those cases, and whether they are possible is open to question. Since the problems covered in this paper do not depend on there being an infinite number of alternatives, I shall not discuss this issue further here.

[8] Rabinowicz (2000, p. 124).

other things being equal, there does not seem to be anything irrational in violating the monetary dominance principle. And, if one prefers to be richer other things being equal, the irrationality in violating the monetary dominance principle seems to merely consist in that one thereby violates the diachronic dominance principle.[9]

The aim of the money-pump argument for transitivity is to show that anyone who violates transitivity is in some possible situation (with a finite number of alternatives) forced to violate one of these dominance principles.[10] Amos Tversky presents the argument as follows:

> Transitivity [...] is one of the basic and the most compelling principles of rational behavior. For if one violates transitivity, it is a well-known conclusion that he is acting, in effect, as a "money-pump." Suppose an individual prefers y to x, z to y, and x to z. It is reasonable to assume that he is willing to pay a sum of money to replace x by y. Similarly, he should be willing to pay some amount of money to replace y by z and still a third amount to replace z by x. Thus, he ends up with the alternative he started with but with less money.[11]

In this example, the agent can make four alternative sequences of choices. First, he can turn down all swaps and end up with x. Second, he can accept just one swap and end up with y. Third, he can accept just two swaps and end up with z. Fourth, he can accept all of the swaps and end up with x but with less money than if he had turned down all swaps. For each of these sequences of choices, there is an alternative sequence that is preferred to it. So, regardless of which of these sequences of choices the agent makes, he **[p. 63]** violates the diachronic dominance principle.[12] Thus the agent is in this situation forced to violate this principle.[13]

[9] See Gustafsson (2013b, p. 462). Note also that the original money-pump example by Davidson et al. (1955, pp. 145–146) illustrates a non-monetary version of the dominance principle, namely, the principle that 'a rational choice is one which selects an alternative to which none is preferred'.

[10] Hence the point of the money-pump argument is not, as Parfit (2011, p. 128) claims, to show that non-transitive preferences can have bad effects. So his objection that non-transitive preferences can also have good effects misses its mark.

[11] Tversky (1969, p. 45).

[12] Note that the agent would be forced to violate the diachronic dominance principle even if the swaps were free and there were no third offer to swap back to x. The monetary part of the money-pump argument makes the argument more dramatic but is inessential.

[13] Some people object that the agent can avoid being money pumped—that is, accepting all the swaps—if he solves his decision problem by using backward induction; see, for example, McClennen (1990, sec. 10.2) and Rabinowicz (1995, pp. 592–596; 2000, pp. 136–140). If the agent reasons by backwards induction, he will just accept one swap and end up with y. This is because he can predict that were he to accept the second swap he would also accept the third swap—since he prefers x to z—and then he would end up with x to which he prefers y. But, while the agent can avoid exploitation if he reasons

1. Permitted Incompleteness

Tversky's version of the money-pump argument does not, however, take into account the possibility of incompleteness, that is, the possibility of preferential gaps where neither of two alternatives is at least as preferred as the other.[14] If we allow that rational preferences may include preferential gaps, the agent could instead violate transitivity by preferring z to y and y to x while having a preferential gap between x and z. The trouble is that the agent can then reject the swap from z to x without violating any of the dominance principles, because there is no alternative sequence of choices that is preferred to the one where he ends up with z.

Here, one might object that, even though the standard money-pump argument fails if preferences are rationally permitted to be incomplete, there is a variant of the money-pump argument that works, namely, the non-forcing money-pump argument. The basic premise of this variant is that, if one violates one of the dominance principles by making a certain sequence of choices, then some choice in that sequence of choices is rationally forbidden. If the agent who prefers z to y and y to x while having a preferential gap between x and z makes the sequence of choices where he accepts all swaps in Tversky's example, then the agent violates the diachronic dominance principle. The first two choices in this sequence of choices—that is, the swap from x to y and the swap from y to z—are not only rationally permitted but also rationally required, since, if one turns down these swaps, one violates the diachronic dominance principle. So, if there is a rationally forbidden choice in this sequence, it is the last one, where x is chosen over z. If the agent can avoid violating some requirement of rationality in this case, it cannot be rationally forbidden to turn down the swap from z to x, since any other sequence of choices violates the diachronic dominance principle. Hence we have that, in the last choice between x and z, choosing x is rationally forbidden but choosing z is not rationally forbidden. Which brings us to the crucial transfer premise in

in this way, he would still violate the diachronic dominance principle. One might object here that the diachronic dominance principle is questionable, since one does not choose sequences of choices (unless one is resolute). And so, by performing a sequence of choices to which an alternative sequence is preferred, one need not make any irrational choice; see, for example, Hedden (2015). But, if this objection to the diachronic dominance principle works, it should also work as an objection to the monetary dominance principle. Hence it does not show that we have any more reason to take exploitability as a sign of irrationality than non-monetary violations of the diachronic dominance principle.

[14] Moreover, Tversky's version of the money-pump argument does not take into account the possibility of indifference. If one violates transitivity by preferring z to y and y to x while being indifferent between x and z, one may reject the swap from z to x without violating any of the dominance principles. See Gustafsson (2010) for a discussion of this problem.

the non-forcing money-pump argument: In a choice between x and z, if the agent has a preferential gap between **[p. 64]** x and z and choosing z is not rationally forbidden, then choosing x is also not rationally forbidden. Given this transfer premise, we have a contradiction. We need to assess, however, whether we should accept this transfer premise. This kind of transfer principle has mainly been discussed and defended in the context of value incomparability rather than in the context of preferential gaps, but the same considerations about transfer principles seem to apply in both of these closely related contexts, changing what needs to be changed.

So, to assess this transfer premise, it will help to consider a similarly structured non-forcing money-pump argument against value incomparability. The idea behind this argument is similarly that, if there were incomparable alternatives, there would be cases where agents can, without making a single wrong choice, make a sequence of choices that is worse than an alternative sequence of choices. John Broome writes:

> Suppose two careers are open to you: a career in the army and a good career as a priest. Suppose they are incommensurate in their goodness. Then choosing either would not be wrong. You have to choose without the guidance of reason, and suppose you choose the army: you commit yourself to the army career, and give up the chance of a good career in the church. In doing so you are doing nothing wrong. But then suppose another opportunity comes up to join the church, this time in much worse conditions. You now face a choice between the army or a much less good career as a priest. Suppose these two, also, are incommensurate. Choosing either would not be wrong. You have to choose without the guidance of reason. Suppose this time you choose the church. Once again you do nothing wrong. But though you have not acted wrongly in either of your choices, the effect of the two together is that you end up with a much worse career in the church than you could have had. Surely rationality should be able to protect you from this sort of bad result; surely there is something irrational in what you have done. Yet apparently neither of your decisions was irrational. This is puzzling.[15]

The key premise in Broome's argument is that choosing either of two incomparable alternatives is not wrong. An underlying assumption behind money-pump arguments is that, in situations with a finite number of alternatives, rational agents can never be forced to make an irrational choice. So, in this context, we may plausibly assume that, in a choice between two incomparable alternatives, at least one of them is neither irrational nor wrong to choose. Yet Broome's argument needs the questionable, stronger

[15] Broome (1999, p. 156). Chang (1997, p. 11) presents a similar case.

premise that it is not wrong to choose either of two incomparable options over the other.

Martin Peterson claims that this last premise in Broome's argument can be supported by the following transfer principle:

> *The transfer principle*
> If two alternatives are incomparable and choosing one of them is not wrong, then choosing the other is also not wrong.[16]

[p. 65] Given this principle, it follows that, in each choice between incomparable alternatives in Broome's example, it is not wrong to choose either alternative—or, at least, it follows if we, as seems plausible, accept that there has to be at least one alternative in these choices which it is not wrong to choose.

There are, however, compelling counter-examples to the transfer principle. Suppose that, in Broome's example, you were offered a choice between the good priest career, the less good priest career, and the army career. It seems then that it would be wrong to choose the less good priest career and not wrong to choose the army career. Choosing the less good priest career seems wrong since there is a better alternative, that is, the good priest career. But the army career is not worse than any alternative, so choosing it might plausibly be not wrong. Hence, in this situation, it seems that there are two incomparable alternatives such that it is wrong to choose the first but not wrong to choose the second.

In response to this alleged counter-example, Peterson argues that the transfer principle is supported by the following supervenience principle:

> *The normative-supervenience principle*
> The normative statuses of the available alternatives in a situation are determined by the evaluative ranking of those alternatives.[17]

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[16] Peterson (2013, pp. 133–135) states his transfer principle in terms of permissibility: If one of two incomparable alternatives is permissible, then the other is also permissible. But since what is at stake in Broome's argument is whether you did not do anything wrong in either of your choices, it fits our purposes better to state the principle in terms of alternatives not being wrong. My revision should not misrepresent Peterson's (2013, p. 130) views, because he holds that alternatives are either permissible or forbidden. Peterson (2007, pp. 510–512) presents a less general version of the argument, which only targets certain kinds of incomparability.

[17] Peterson (2013, p. 135) writes:

> I shall assume that normative properties supervene on evaluative properties in the following sense: an evaluative ranking of a set of objects, in which all objects are listed from the best to the worst, determines the normative properties of the objects. Hence, if two objects are, for instance, on a par on the evaluative scale, and therefore have the same normative properties, the addition of a third object cannot affect the normative property of one of the original objects, unless that of the other object is also affected.

Yet this principle does not rule out that whether an alternative has a certain normative status depends on whether it is worse than another alternative. Hence it does not rule out that an alternative is wrong if and only if there is a better alternative. So this principle does not rule out that, in a choice between the army career and the less good priest career, choosing the less good priest career is not wrong (because there is no better alternative) but, in a choice between the army career and the two priest careers, choosing the less good priest career is wrong (because then there is a better alternative). Thus the normative-supervenience principle does not block the counter-example to the transfer principle.

There is, however, another way to avoid counter-examples of this kind. Instead of the transfer principle above, one could rely on the following, more restricted variant:

> *The restricted transfer principle*
> In a choice between two incomparable alternatives, if choosing one of them is not wrong, then choosing the other is also not wrong.

[p. 66] Since the above counter-example to the first transfer principle involves a choice between three alternatives, it does not apply to this revised principle. Besides, this restricted transfer principle is supported by the normative-supervenience principle. The normative-supervenience principle yields that, in a choice between two incomparable alternatives, the alternatives have the same normative status, because incomparability is symmetric. So we get that, if choosing one of the alternatives is not wrong, choosing the other alternative is also not wrong.

But the normative-supervenience principle might plausibly be rejected. While it is plausible that evaluative properties should guide our choices when one of two alternatives is better than the other, it is less plausible when the alternatives are incomparable. When two alternatives are incomparable, their evaluative properties neither favour them equally nor favour one of them over the other. That is why it makes sense that other factors could help determine the incomparable alternatives' normative statuses. Some other factors that might plausibly come into play are one's previous choices, which could be relevant for preventing violations of the diachronic dominance principle. So we can plausibly reject the normative-supervenience principle. And, without that principle, neither

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One could perhaps instead read Peterson as proposing here that the normative statuses of the available alternatives in a situation are determined by the evaluative ranking of all possible alternatives in the domain, that is, not just the available alternatives. But that reading does not fit the last part of the quote, where the mere addition of alternatives might affect the normative statuses of the original alternatives. Nevertheless, my objections to the supervenience principle also apply to this alternative supervenience principle. See also Bykvist (2003, p. 30) for a similar principle.

of the transfer principles has any support and can hence plausibly be rejected too. Thus we need not accept that, in a choice between two incomparable alternatives, it is not wrong to choose either of the alternatives. And then we can reject Broome's argument.[18]

By an analogous line of reasoning, we can plausibly reject the transfer premise in the non-forcing money-pump argument for transitivity. And then we can also reject that argument. Thus it seems that neither the non-forcing nor the standard money-pump argument for transitivity works if preferences are rationally permitted to be incomplete.

2. Required Completeness, Permitted Indeterminacy

Preferences, however, are perhaps rationally required to be complete after all. The standard, alleged examples of rational preferential gaps are perhaps, as mentioned in the introduction, merely examples of indeterminacy. I shall argue that, while the standard version of the money-pump argument fails if preferences are rationally permitted to be indeterminate, a new version works.

For the discussion of indeterminate preferences, I shall adopt a version of supervaluationism.[19] Let the sharpenings of a statement be the admissible ways of making it precise. Then, according to a supervaluationism, a statement is indeterminate if and **[p. 67]** only if it comes out as true on some of its sharpenings and as false on some of its other sharpenings. A statement is true if and only if it comes out as true on all of its sharpenings. And a statement is false if and only if it comes out as false on all of its sharpenings. Admittedly, supervaluationism is a contentious theory; it is mainly adopted here to ease the presentation of the discussion, which should not depend crucially on the theory's more controversial traits.[20]

An initial source of worry is that Tversky's version of the money-pump argument does not work if indeterminate preferences are rationally

[18] If we allow that whether an alternative is wrong in a situation can depend in part on what choices the agent would go on to make in the future, then there is another problem with Broome's argument—even if we were to accept these transfer principles. Suppose that in the first situation, where you have a choice between the army career and the good church career, it is only wrong to choose the army if you would switch to the less good church career. Suppose that you chose the army in first situation and that you face the second situation, where you have a choice between the army and the less good church career. Finally, suppose that, in the second situation, there is at least one choice that is not wrong. Then, given either of the transfer principles, we have that choosing the less good church career is not wrong in the second situation. The problem is that, if you were to choose the less good church career, then your choice in the first situation would be wrong, even though you would not make a wrong choice in the second situation.

[19] See Fine (1975) and Keefe (2000, ch. 7).

[20] See, for example, the objections to supervaluationism in Williamson (1994, ch. 5).

permitted. To see this, consider an agent such that it is indeterminate which of the following four sets of preferences is the agent's preferences:

(1) *b* is preferred to *c*, *c* is preferred to *d*, *d* is preferred to *b*, and *a* is preferred to each of *b*, *c*, and *d*.

(2) *a* is preferred to *c*, *c* is preferred to *d*, *d* is preferred to *a*, and *b* is preferred to each of *a*, *c*, and *d*.

(3) *a* is preferred to *b*, *b* is preferred to *d*, *d* is preferred to *a*, and *c* is preferred to each of *a*, *b*, and *d*.

(4) *a* is preferred to *b*, *b* is preferred to *c*, *c* is preferred to *a*, and *d* is preferred to each of *a*, *b*, and *c*.

Furthermore, suppose that it is determinate that the agent has the preferences in one of (1), (2), (3), and (4). Each of (1)–(4) violates not only transitivity but also the weaker requirement of acyclicity. Preferences are acyclic if and only if, for all $x_1, x_2, x_3, \ldots$, and x_n, if x_1 is preferred to x_2, x_2 is preferred to $x_3, \ldots$, and x_{n-1} is preferred to x_n, then x_n is not preferred to x_1. With (1)–(4) being the sharpenings of the agent's preferences, we have that it is determinate that the agent satisfies completeness, because all four of these sharpened preferences are complete. Yet, since all four of these sharpened preferences violate transitivity, it is determinate that the agent violates transitivity.[21] Therefore, since the money-pump argument for transitivity is supposed to show that anyone who violates transitivity is, in some possible situation, **[p. 68]** forced to violate one of the dominance principles, it should be able to do so for this agent.[22]

But Tversky's version of the money-pump argument does not work for this violator of transitivity. Suppose, for example, that the agent starts with *a*. Then we cannot assume that he is willing to pay a sum of money

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[21] Aldred (2007, p. 381) discusses an example where there is a vague property *P* and an agent prefers *x* to *y* if and only if *x* is *P* and *y* is not *P*. And, if the agent does not prefer one of *x* and *y* to the other, she is indifferent between them. Suppose then that it is determinate that *a* is *P*, determinate that *b* is not *P*, and indeterminate whether *c* is *P*. In a case like this, Aldred claims that the agent's preferences are non-transitive, because the agent prefers *a* to *b* but is indifferent between *b* and *c* and between *a* and *c*. This, however, does not seem to follow. If the agent prefers, for example, *a* to *c* if and only if *a* is *P* and *c* is not *P*, and it is indeterminate whether *a* is *P* and *c* is not *P*, then it should be indeterminate whether she prefers *a* to *c*. In this way, it could be indeterminate which of the following two sets of preferences is the agent's preferences:

(I) *a* is preferred to *b*, *a* is preferred to *c*, and *b* is indifferent to *c*.

(II) *a* is preferred to *b*, *c* is preferred to *b*, and *a* is indifferent to *c*.

But this is consistent with that it is determinate that the agent's preferences are transitive, since they are transitive in both (I) and (II).

[22] Since the agent in this example violates not only transitivity but also acyclicity, it is also a challenge for the money-pump argument for acyclicity.

to replace *a* with any of the other alternatives, because it is indeterminate whether *a* is preferred to every other alternative. Thus it is not determinate that the agent violates the synchronic dominance principle if he refuses to swap from *a*. It is also not determinate that the agent violates the diachronic dominance principle if he sticks with *a*, since it is indeterminate whether *a* is optimal. And the same holds, changing what needs to be changed, if the agent starts with one of the other alternatives. So Tversky's method cannot establish that this agent who determinately violates transitivity is, in some possible situation, forced to determinately violate one of the dominance principles.[23]

If the aim of the money-pump argument for transitivity is to show that any agent who violates transitivity is in some possible situation forced to violate one of the dominance principles, it should be able to show that any agent who *determinately* violates transitivity is in some possible situation forced to *determinately* violate one of the dominance principles. Tversky's version of the argument does not meet this desideratum.

For a last attempt at amending Tversky's money-pump argument as an argument for at least acyclicity, note that, if it is determinate that one's preferences violate acyclicity, it is determinate that one can be exposed to a situation in which one violates the dominance principles. In each sharpening, it can be shown by Tversky's method that there is a possible situation where one is forced to violate the diachronic dominance principle. In this manner, we can show that it is determinate that there is a possible situation where the above agent is forced to violate the diachronic dominance principle.

But one might object that this version of the argument only shows that, if it is determinate that one's preferences violate acyclicity, it is determinate that there is a possible situation where one is forced to violate the

[23] One might wonder why this cannot be shown with the simpler example of an agent such that it is indeterminate which of the following two sets of preferences is the agent's preferences:

(III) *a* is preferred to *b*, *b* is preferred to *c*, and *c* is preferred to *a*.

(IV) *a* is preferred to *c*, *c* is preferred to *b*, and *b* is preferred to *a*.

And it is determinate that the agent has the preferences in one of (III) and (IV). Although we cannot assume that this agent needs to accept each swap in Tversky's version of the money-pump argument in order to avoid violating the synchronic dominance principle, the agent is still forced to determinately violate the diachronic dominance principle. This is because in both (III) and (IV) there is, for each of the available sequences of choices in this situation, an alternative sequence that is preferred to it. So, whichever of the available sequences of choices the agent makes, the agent will determinately violate the diachronic dominance principle. Similarly, given a choice between all three of *a*, *b*, and *c*, the agent is forced to violate the synchronic dominance principle. Neither of these methods work in the more complicated example with (1)–(4).

diachronic dominance principle. It does not show that, if one determinately violates acyclicity, there is a possible situation where one is forced to determinately violate one of the dominance principles. Remember that the underlying idea behind money-pump arguments is that a requirement is a requirement of rationality if violating the requirement will in some possible situation force a violation of **[p. 69]** some other requirement of rationality. It might hence seem unwarranted to conclude here that it is determinate that acyclicity is rationally required, since we only have that determinately violating acyclicity at most forces one in some possible situation to an indeterminate violation of dominance.

I shall argue, however, that, for each agent who determinately violates transitivity, there is some possible situation where the argent is forced to determinately violate one of the dominance principles.[24] In this new variation of the money-pump argument, we consider lotteries in which the possible outcomes are the options over which one has non-transitive but indeterminate preferences.[25] The new variation requires one more dominance principle, namely, the following principle for preferences over lotteries:

> *The lottery dominance principle*
> It is rationally required that, if there is a partition of states such that it is independent of lotteries L' and L'' and relative to it there is at least one positively probable state where the outcome of L'' is preferred to the outcome of L' and no state where the outcome of L'' is not at least as preferred to the outcome of L', then L'' is preferred to L'.[26]

I leave open here whether the relevant kind of independence between lotteries and states is evidential or causal.[27]

Suppose that an agent determinately violates transitivity but satisfies completeness. Let then A be a set of outcomes such that for each sharpening of the agent's preferences there are three outcomes x, y, and z in A such that it is true in the sharpening that x is at least as preferred as y, y is at least as preferred as z, and z is preferred to x. Let S be a partition of positively probable states such that there are equally many states as

[24] One might think that the following example shows that this claim is false: Suppose that an agent prefers x to each of y, z and w while she violates transitivity by preferring y to z, z to w, and w to y. If this agent can simply choose x, she can obviously avoid violating the dominance principles. But this is consistent with my claim; it only says that there is *some* situation where the agent is forced to violate one of the dominance principles. For example, this agent is forced to violate the synchronic dominance principle in a situation where she must choose one of y, z and w.

[25] This approach generalizes my approach in Gustafsson (2010, pp. 255–256).

[26] See, for example, Savage (1951, p. 58) and Nozick (1969, p. 118).

[27] See Joyce (1999, pp. 150–151).

there are outcomes in A and, moreover, the states in S are independent in the relevant sense (causally or evidentially) from the lotteries which will be offered next. Offer the agent a choice between all permutations of lotteries such that each element in A is an outcome for one state in S.[28] In each sharpening of the agent's preferences, we have that whichever lottery L' the agent chooses there will be another available lottery L'' that dominates L' in the sense that, for every state, the outcome of L'' is at least as preferred as the outcome of L' and, for at least one state, the outcome of L'' is preferred to outcome of L'. To see this, note first that, in each sharpening of the agent's preferences, there will be at least one group of three outcomes in A over which the agent's preferences constitute a violation of transitivity. Since we are working under the assumption that rational preferences are complete, we have then that in every lottery there are three positively probable **[p. 70]** outcomes x, y, and z such that x is at least as preferred as y, y is at least as preferred as z, and z is preferred to x. But then, for each available lottery L', there is an available permuted lottery L'' that has the same outcome as L' in all states except that, in the state where L' has outcome x, L'' has outcome z; in the state where L' has outcome y, L'' has outcome x; and, in the state where L' has outcome z, L'' has outcome y. We have in every state that the outcome of L'' is at least as preferred as the outcome of L', and we have in one state that the outcome of L'' is preferred to the outcome of L'. Hence L' is dominated by L''. Thus it is determinate that, if the agent satisfies the lottery dominance principle, then whichever lottery the agent chooses there is another available lottery which is preferred to the chosen lottery. Hence we have a situation where it is determinate that the agent violates either the synchronic dominance principle or the lottery dominance principle.

Consider for example an agent such that it is indeterminate which of the following two sets of preferences is the agent's preferences:

(5) a is preferred to b, b is indifferent to c, and c is indifferent to a.

(6) a is preferred to c, c is indifferent to b, and b is indifferent to a.

And suppose that it is determinate that the agent has the preferences in one of (5) and (6). For this agent, we can let A be the set $\{a, b, c\}$ and let S be a partitioning with three positively probable states, which are independent of the lotteries offered next. We offer the agent a choice between all permutations of lotteries such that each element in A is an

[28] If one, for some reason, thinks that money-pump arguments are more convincing if they just rely on pairwise choices, one could amend the approach as follows: Instead of a single choice, offer the agent a series of pairwise choices such that each of these lotteries is the outcome of one of the available sequences of choices and each of these sequences has one of these lotteries as its outcome. Then, whichever of these sequences of choices the agent makes, the agent will violate the diachronic dominance principle.

outcome for one state in S. Let $[x_1, x_2, x_3]$ be the lottery that, relative to the three states in S, has outcome x_1 in the first state, outcome x_2 in the second, and outcome x_3 in the third. Hence the agent has a choice between the lotteries $[a, b, c]$, $[a, c, b]$, $[b, a, c]$, $[b, c, a]$, $[c, a, b]$, and $[c, b, a]$. In (5), any one of the available lotteries is dominated by the one of the other lotteries where the outcomes are distributed so that it has outcome c in the state where the first lottery has outcome a, it has outcome a in the state the first lottery has outcome b, and it has outcome b in the state where the first lottery has outcome c. Similarly, in (6), any one of the available lotteries is dominated by the one of the other lotteries where the outcomes are distributed so that it has outcome b in the state where the first lottery has outcome a, it has outcome c in the state where the first lottery has outcome b, and it has outcome a in the state where the first lottery has outcome c. So we have, for example, that it is determinate that lottery $[a, b, c]$ is dominated by another available lottery, because $[a, b, c]$ is dominated in (5) by $[c, a, b]$ and in (6) by $[b, c, a]$.[29] We have that every available lottery is dominated in each sharpening of the agent's preferences. Hence it is determinate that, if the agent satisfies the lottery dominance principle, then, for each available lottery, there is another available lottery which is preferred to it. Thus, in this situation, it is determinate that the agent whose sharpened preferences are (5) and (6) is forced to violate either the lottery dominance principle or the synchronic dominance principle.

As another example, consider again the agent whose sharpened preferences are (1)–(4). For this agent, let A be the set $\{a, b, c, d\}$ and let S be a partitioning with four **[p. 71]** positively probable states, which are independent of the lotteries offered next. Like before, we offer the agent a choice between all permutations of lotteries such that each element in A is an outcome for one state in S. This time, since we are permutating four elements, the agent has a choice between twenty-four—that is, the factorial of four—lotteries. Let $[x_1, x_2, x_3, x_4]$ be the lottery that, relative to the four states in S, has outcome x_1 in the first state, outcome x_2 in the second, outcome x_3 in the third, and outcome x_4 in the fourth. With the same method as before, we can show that, in every sharpening, any one of the available lotteries is dominated by the one of the other lotteries. For example, we have that it is determinate that lottery $[a, b, c, d]$ is dominated by another available lottery, because $[a, b, c, d]$ is dominated

[29] In (5), $[a, b, c]$ is dominated by $[c, a, b]$, since in the second state the outcome of $[c, a, b]$ is preferred to the outcome of $[a, b, c]$ and, in every other state, the outcome of $[c, a, b]$ is at least as preferred as the outcome of $[a, b, c]$. Similarly, in (6), $[a, b, c]$ is dominated by $[b, c, a]$, since in the third state the outcome of $[b, c, a]$ is preferred to the outcome of $[a, b, c]$ and, in every other state, the outcome of $[b, c, a]$ is at least as preferred as the outcome of $[a, b, c]$.

in (1) by $[a, d, b, c]$, in (2) by $[d, b, a, c]$, in (3) by $[d, a, c, b]$, and in (4) by $[c, a, b, d]$. Hence we have a situation where it is determinate that the agent whose sharpened preferences are (1)–(4) is forced to violate either the synchronic dominance principle or the lottery dominance principle.

In this manner, we can show that, for any agent with complete preferences who determinately violates transitivity, there is a possible situation where that agent is forced to determinately violate one of the dominance principles. Hence we have a version of the money-pump argument for transitivity that works if preferences are rationally permitted to be indeterminate yet rationally required to be complete. Or, at least, we have a version of the argument that can overcome the problems posed by indeterminacy in rationally permitted preferences. So, on the one hand, the money-pump argument for transitivity fails if preferences are rationally permitted to be incomplete. But, on the other hand, it might still work if preferences are rationally permitted to be indeterminate and rationally required to be complete.

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