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Karin Enflo *Measures of Freedom of Choice*, Uppsala University, 2012, 253 pp., ISBN 978-91-506-2296-6

Reviewed by Johan E. Gustafsson

When does a set of alternatives offer more freedom of choice than another? A straightforward answer is the cardinality-based measure, according to which a set offers more freedom of choice than another set if and only if it has more alternatives than the other set. Yet this answer yields implausible results in some cases. A set with very similar alternatives might seem to offer less freedom of choice than a set with fewer but less similar alternatives. This suggests that an adequate measure of freedom of choice should somehow take into account the similarity of alternatives (Pattanaik and Xu, 1990, pp. 389–390). How the similarity of alternatives should be taken into account, however, is unclear.

Karin Enflo's *Measures of Freedom of Choice* attempts to shed light on this problem. The book has three parts. The first part introduces the concept of freedom of choice and a framework for its measurement. The second part surveys the literature on measures of freedom of choice and accepts a number of conditions for such measures. Together these first two parts provide an ambitious and useful overview of the extensive literature on this topic. Finally, the third part presents the characterization result that a certain class of measures are the only measures that satisfy all the accepted conditions.

1. The ratio-root measures

The main claim of the book is that we should accept a class of measures of freedom of choice called the ratio-root measures. Let A be a set of alternatives. Let n be the number of alternatives in the set. And let $d(x, y)$ be the degree of dissimilarity between alternatives x and y. Then, according to the ratio-root measures, freedom of choice can be measured by a function F as follows (p. 195):

Ratio root measures:

i) For $n \leq 1$, $F(A, d) = 0$.

ii) For $n > 1$, $F(A, d) = \frac{1}{n-1} \sum_{i=1, i \neq j}^{n} \sum_{j=1}^{n} (d(x_i, x_j))^r$, with ½ ≤ $r < 1$.

This rather complicated formula is meant to say that the score in terms of freedom of choice of a set of alternatives *A* is calculated as follows: If *A* has less than two **[p. 88]** alternatives, then the score is 0. If *A* has two or more alternatives, then we add up, for each ordered pair of distinct alternatives in *A*, the dissimilarity between the alternatives in this pair raised to *r*, where *r* is at least as great as ½ and lesser than 1. Since the exact value of *r* is left open, the formula describes a family of measures. According to Enflo, the attractive feature of the ratio-root measures is that they are the only measures that satisfy a group of adequacy conditions which she argues should be satisfied by any acceptable measure.

Yet a problem with the ratio-root measures is that they are overly sensitive to the number of alternatives in a set. To illustrate this, we shall follow Robert Sugden (1998, p. 318) and consider choices between office temperatures. Suppose that the degree of dissimilarity between office temperatures is equal to their difference in temperature measured in degrees Celsius. Consider then the following sets of alternatives:

$A = \{16\ °C,\ 22\ °C,\ 28\ °C\}.$

$B = \{16\ °C,\ 16.1\ °C,\ 27.9\ °C,\ 28\ °C\}.$

Set *A* seems to offer more freedom of choice than *B*. As well as offering a very cold office temperature (that is, 16 °C) and a very warm office temperature (that is, 28 °C), *A* offers a moderate alternative at 22 °C, whereas *B* only offers very cold and very warm office temperatures. The trouble is that according to all ratio-root measures *B* offers more freedom of choice than *A*. Choose, for example, $r = ½$. Then we have that *B* scores approximately 9.6 in terms of freedom of choice whereas *A* just scores approximately 8.4. Moreover, all ratio-root measures would yield that *B* offers more freedom of choice than *A* even if the 16.1 °C alternative were replaced in *B* by an alternative arbitrarily more similar to 16 °C or if the 27.9 °C alternative were replaced in *B* by an alternative arbitrarily more similar to 28 °C. Hence it seems that the ratio-root measures do not adequately measure freedom of choice; they overrate the increase of freedom of choice resulting from the addition of a new alternative to a set when the new alternative is very similar to one of the original alternatives. In other words, the ratio-root measures suffer from the same the problem with similar alternatives as the simple cardinality-based measure. Hence they do not solve the problem that motivated taking the dissimilarity of alternatives into account in the first place.

Discussing this problem, Enflo admits that she has no solution. She introduces a further condition, the extreme-limited-growth condition, which neatly rules out counter-examples of the above type (p. 199). None of the ratio-root measures, however, satisfies this condition. In response to this predicament, Enflo writes that **[p. 89]**

> There may be a measure that satisfies the *Extreme limited growth condition* without failing to satisfy the most important of the ten accepted conditions. However, before such a measure is found, it seems better to accept the *Ratio root measures* rather than to accept the *Extreme limited growth condition*. (p. 200)

But, if a counter-example shows that the ratio-root measures are implausible and other existing accounts are also implausible, a more fitting response would be to suspend judgement and continue the search for a plausible account. And, since one cannot accept a group of conditions that characterize the ratio-root measures without being vulnerable to the above counter-example, the example seems to call for a second look at these conditions and the arguments for accepting them.

2. The spread condition

One of the conditions in Enflo's characterization of the ratio-root measures is the spread condition. To understand Enflo's statement of the condition, we need to introduce some more of her notation. Let the metric space X_d be the pair of the set X of all possible alternatives and the dissimilarity function d. Let $P(X_d)$ be the set of all subspaces of X_d. And let $\sum(A_d)$ be the sum total of the degrees of dissimilarity between the alternatives in A.

> *The Spread condition*: For a measure of freedom of choice F, and all metric choice sets $A_d, B_d \in P(X_d)$ such that $|A| = m$ and $|B| = n$ and $\sum(A_d) = \sum(B_d) = M$, where each non-zero distance $d_{Ai} = M/(m(m-1))$ and each non-zero distance $d_{Bi} = M/(n(n-1))$, if $m < n$, then A *offers strictly more freedom of choice than* B, and thus $F(A) > F(B)$. (p. 168)

This is meant to say that, if (i) a set of alternatives A has the same sum total of the degrees of dissimilarity between its alternatives as another set of alternatives B and (ii) A has fewer alternatives than B and (iii) in both A and B each alternative is equally dissimilar to all other alternatives in the same set, then A offers more freedom of choice than B. Enflo illustrates the underlying intuition as follows:

> let us suppose that there are only two options in *A* and that the difference between them is 45. In *B*, there are 10 options and the difference between each pair of options is 1. The sum of differences is the same in each case: 90. Here, most people would judge *A* to be the more diverse set. For example, a set of ten left-wing parties would not be judged as more diverse than a set of one left-wing party and one right-wing party. (p. 168)

I am not as compelled to make a judgement in this case as apparently most people are. First of all, it takes some effort to imagine what it would be like to have a choice between ten left-wing parties such that each party is 1 unit dissimilar from all the other parties. One way to do this is to imagine an eleventh **[p. 90]** party from which the ten parties in *B* respectively differ a little bit in ten independent respects of their policies, such as foreign relations, health care, the environment, and so on. While the parties in *B* are all left-wing parties, they still seemingly offer a lot of variation, varying in different ways in ten independent respects. It is not clear to me that this cannot make up for the alternatives' in *B* being pairwise more similar than those in *A*, whose alternatives we may assume only differ in one of these ten respects.

Yet, even if we were to agree that the spread condition yields the right result in this case, this would not provide much support for the condition since it might fare worse in other cases. As it happens, I think the spread condition yields a less plausible result in a variation of the case. Suppose instead that *A* offers the following alternatives: the mainstream centre party or the very radical death-for-all party. Moreover, suppose that these alternatives are 45 units dissimilar. And suppose that *B* instead offers ten variations the centre party, which respectively differ a little bit from the centre party in ten different independent respects, so that each party is 1 unit dissimilar from all the other parties. Like before, suppose that the respects in which the ten parties differ are things about which reasonable people disagree, like foreign relations, health care, the environment, and so on. It seems then to me that the selection in *A* between the centre party and death offers very little freedom of choice, since there is only one reasonable choice; whereas the selection in *B* between ten parties that push the mainstream centre party in ten different directions offers much more freedom of choice. According to the spread condition, however, *A* offers more freedom of choice than *B*.

In addition to the above example, Enflo provides one further reason for accepting the spread condition. She writes:

> In cases where the difference in magnitude and cardinality is small, the condition may perhaps be opposed. Looking at the most similar comparisons, someone may think that *A*, with three options

> and the distance vector $\mathbf{d_A} = (1, 1, 1, 1, 1, 1, 0, 0, 0)$, is no less diverse than B with two options and the distance vector $\mathbf{d_B} = (3, 3, 0, 0)$. But this would be rather odd. The extra option in A hardly makes up for the loss of diameter. As the difference between the diameters increases between the larger set and the smaller set, it gets increasingly strange to continue to insist that the larger set with very similar options is as diverse as the smaller set with very different options. Because of this, I shall accept the *Spread condition*. (p. 168)

Here, the diameter of a set is the maximum degree of dissimilarity between two alternatives in the set. This argument, which is apparently decisive for Enflo's acceptance of the spread condition, concerns just the spread condition's implications in cases where the difference in magnitude and cardinality is small. Hence, even if successful, this reasoning does not support accepting the spread condition's implications in general. Moreover, that '[t]he extra option in A hardly makes up for the loss of diameter' is a somewhat question-begging reply to the objection that A seems no less diverse than B. **[p. 91]**

3. The proportional-growth condition

Another condition in Enflo's characterization of the ratio-root measures is the proportional-growth condition, which is stated as follows:

> *The Proportional growth condition*: For a measure of freedom of choice F, and any metric choice set $A_d \in P(X_d)$ such that $|A| \geq 2$, if $|A| = n$ and each non-zero distance $d_{Ai} = 1$, then $F(A) = n$. (p. 196)

This is meant to say that, if a set of alternatives has at least two alternatives and each of its alternatives is 1 unit dissimilar from every other alternative in the set, then the set scores n in terms of freedom of choice. This condition might seem implausible since the unit for measurements of freedom of choice is arbitrary. Enflo initially shares this worry. She writes that

> it does not matter what exact numerical value is assigned to a choice set when the number of options is n and the distances between all pairs of options are 1, as long as the correct ratios between the degrees of freedom of choice offered by different sets is preserved. But since the exact numerical value does not matter, it also does not matter that a set is assigned the number n when the number of options is n and their distances are 1. So the condition is acceptable. (p. 197)

Yet the above worry is not that it is arbitrary to score these sets a score of *n* rather than some other score. The worry is that, if a measure of freedom of choice that satisfies the proportional-growth condition were acceptable, then another measure should also be acceptable, which—holding other things fixed—simply doubles all the scores of the first measure. But the second measure would not satisfy the proportional-growth condition.

Moreover, even if we can sidestep this worry about the arbitrariness of units, we still have not been given any reason to accept the proportional-growth part of the condition. While it is plausible that the amount of freedom of choice offered grows as we move to larger sets of alternatives in which the alternatives are 1 unit dissimilar from each other, I see no cogent reason to accept that this growth is exactly proportional to the number of alternatives in the sets, rather than growing in some other manner.

Summing up, it seems that at least two of the conditions in the characterization of the ratio-root measures are unconvincing. Hence this characterization result does not support the main claim of the book, that is, the claim that we should accept the ratio-root measures. Overall, however, the book provides a stimulating discussion of an important topic. And its thorough overview of the literature should be valuable for further studies in the measurement of freedom of choice. **[p. 92]**

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