

*A Gödelian Ontological Proof with More Plausible Axiological Principles**

Johan E. Gustafsson[†]

ABSTRACT. A drawback of the standard modal ontological proof is that it assumes that it's possible that there is something godlike. Kurt Gödel's ontological proof seeks to establish this possibility with the help of certain axiological principles. The axiological principles he relies on, however, are not very plausible. And the same goes for other Gödelian ontological proofs in the literature. In this paper, I put forward a Gödelian ontological proof that only relies on plausible axiological principles. And I adapt the proof both for constant and varying domains. Nevertheless, the proof still needs the axiom that being godlike is positive in the sense of being a "purely good"-making property.

The standard modal ontological proof for the existence of a godlike being runs as follows: It's possible that something godlike exists. That something godlike exists strictly entails that it's necessary that something godlike exists. Therefore, by standard principles of modal logic, something godlike exists.¹

A drawback of this proof that it assumes that it's possible that there is something godlike. Kurt Gödel's ontological proof seeks to establish this possibility with the help of certain axiological principles we can show that it is possible that there is something godlike.² But the axiological principles he relies on are not very plausible. And the same goes for other Gödelian ontological proofs in the literature. In this paper, I will put forward a Gödelian ontological proof that only relies on plausible axiological principles. And I will adapt the proof for both constant and varying domains.

* Forthcoming in *Ergo*.

[†] I would be grateful for any thoughts or comments on this paper, which can be sent to me at johan.eric.gustafsson@gmail.com.

¹ Hartshorne 1962, p. 51 and Plantinga 1974a, p. 111; 1974b, p. 214.

² Gödel 1987; 1995a. Nonetheless, Anderson (2015, pp. 286–8), who once defended a Gödelian ontological proof (Anderson 1990), prefers Hartshorne's standard modal ontological proof.

Gödel's proof relies on the principle that, for all properties, exactly one of the property and its complement is positive:³

$$(1) \quad \forall\phi(P(\neg\phi) \equiv \neg P(\phi)).$$

Here, $P(\phi)$ states that property ϕ is positive. A *positive* property, is according to Gödel, a property that is 'positive in the moral aesthetic sense'.⁴ He adds that positive properties can be interpreted as perfective properties — that is, “purely good”-making properties.⁵

The right-to-left direction of (1) is implausible. There seems to be neutral properties such that neither the property nor its complementary property is positive.⁶ The property of being male seems neutral and hence not positive, but the property of not being male also seems neutral and not positive.⁷ (If one does not accept that being male is neutral, one could replace being male with some other neutral property that does not seem to be the negation of a positive property.)

Gödel's proof also relies on the following axiological principle:⁸

$$(2) \quad \forall\phi\forall\psi\left(\left(P(\phi) \ \& \ \Box\forall x(\phi(x) \supset \psi(x))\right) \supset P(\psi)\right).$$

³ Gödel 1987, p. 256; 1995a, p. 403. Magari (1988, p. 13), Fitting (2002, pp. 146, 165), and Kovač (2003, p. 572) rely on the same principle. If ϕ and ψ are properties, then $\neg\phi$ is an abbreviation of $\lambda x(\neg(\phi(x)))$, where $\lambda x(f(x))$ is a property an individual y has in virtue of being such that $f(y)$. Likewise, $\phi \vee \psi$ is an abbreviation of $\lambda x(\phi(x) \vee \psi(x))$, and $\phi \ \& \ \psi$ is an abbreviation of $\lambda x(\phi(x) \ \& \ \psi(x))$. See Carnap 1947, p. 3.

⁴ Gödel 1987, p. 257; 1995a, p. 404. Bjørdal (1999, p. 215) defines positivity in terms of godlikeness rather in terms of value — namely, he takes a positive property to be a property such that it's necessary that any godlike being has the property. This does not fit with an axiological interpretation of positivity, since, on Bjørdal's view, tautological properties would be positive (whereas, axiologically, they seem to be neutral rather than positive).

⁵ Gödel 1995b, p. 435. Presumably, we need to restrict positivity to intrinsic properties and to non-limiting properties — that is, properties that are non-limiting in the sense that they do not rule out that one has a basic good-making property to some greater degree. For example, the property of being benevolent to the positive degree n seems to be a good-making property but it rules out being benevolent to any higher degree. I thank Krister Bykvist for this point.

⁶ Anderson 1990, p. 295.

⁷ Gustafsson 2020, p. 232.

⁸ Gödel 1995a, p. 403. Scott (1987, p. 257), Magari (1988, p. 14), Anderson (1990, p. 291), Kovač (2003, p. 572), Maydole (2003, p. 301), Johnson (1999, p. 99; 2004, p. 121), Pruss (2009, p. 347; 2012, pp. 203, 205), and Benz Müller (2020, p. 786; 2022, p. 958) also rely on this principle. Fitting (2002, p. 165) relies on a varying-domain analogue.

Yet this principle is likewise implausible. Let G be the property of being godlike and D be the property of being devil-like. While G seems positive (as we will assume later on), $G \vee D$ (the property of being godlike or devil-like) does not seem positive.⁹ Yet G implies $G \vee D$. So, according to (2), if G is positive, then $G \vee D$ is also positive. But there seems to be no more reason to regard $G \vee D$ as positive than to regard it as negative.¹⁰ (If one does not accept that the magnitude of the negativity of D matches the magnitude of the positivity of G , one could replace G and D with some other pair of positive and negative properties that are alike in magnitude.)

Similarly, tautological properties like $G \vee \neg G$ (the property of being godlike or not godlike) does not seem positive even if one of their disjuncts is positive.¹¹ But, according to (2), if G is positive, then $G \vee \neg G$ is also positive. It seems that, if (2) holds for the logic of positivity, then the following analogous principle should hold for the logic of negativity, where $N(\phi)$ states that property ϕ is negative:

$$(3) \quad \forall\phi\forall\psi\left(\left(N(\phi) \ \& \ \Box\forall x(\phi(x) \supset \psi(x))\right) \supset N(\psi)\right).$$

But, if we accept both (2) and (3), we find that $G \vee D$ is both positive and negative, which conflicts with the following principle:¹²

$$(4) \quad \forall\phi\left(\neg(P(\phi) \ \& \ N(\phi))\right).$$

This principle is plausible given an axiological form of positivity and negativity. It mirrors a standard principle of the logic of value, namely, that nothing is both intrinsically good and intrinsically bad.¹³

Petr Hájek's version of the ontological proof relies on the following axiological principle:¹⁴

$$(5) \quad \forall\phi\forall\psi\left(\left(P(\phi) \ \& \ \Box\forall x(\phi(x) \supset \psi(x))\right) \supset \neg P(\neg\psi)\right).$$

⁹ Hájek 2002, p. 150.

¹⁰ Sobel 2004, p. 122.

¹¹ Sobel 2004, p. 120; 2006a, pp. 406–7; 2006b, p. 286 and van Inwagen 2007, p. 142.

¹² Gustafsson 2020, p. 233.

¹³ Chisholm and Sosa 1966, p. 248

¹⁴ Hájek 2002, p. 156. See also Gödel 1995b, 435 and Cook 2004, p. 106 for similar principles.

This principle states that all positive properties are logically compatible. If any Gödelian ontological proof is to have any success in showing that there is a godlike being that has all positive properties, then those properties need to be logically compatible. But it's not clear why we should accept this principle. The principle has no compelling analogue in the logic of value.

It may seem then that Gödelian ontological proofs need to rely on questionable axiological principles, which does not mirror standard principles in the logic of value. In the following, however, I will show that a Gödelian ontological proof can avoid doing so. We will first consider a constant-domain setting.

1. Constant domain

We adopt the two general formal axiological principles as axioms. The first axiom states that co-entailing properties are alike in positivity:¹⁵

$$(C1) \quad \forall\phi\forall\psi\left(\Box\forall x(\phi(x) \equiv \psi(x)) \supset (P(\phi) \equiv P(\psi))\right).$$

The second axiom states that contradictory properties aren't positive:¹⁶

$$(C2) \quad \forall\phi\neg P(\phi \ \& \ \neg\phi).$$

¹⁵ Gustafsson 2020, p. 235. One may worry that (C1) could fail to hold if we make hyperintensional distinctions among properties (for example, distinguishing being triangular from being trilateral). (See Cresswell 1975, p. 25.) One way to mitigate this worry could be to weaken the principle to just say that, if a first property is logically equivalent to the contradictory property of both having and not having the first property, then those properties are alike in positivity:

$$(C1^*) \quad \forall\phi\left(\Box\forall x(\phi(x) \equiv \phi \ \& \ \neg\phi(x)) \supset (P(\phi) \equiv P(\phi \ \& \ \neg\phi))\right).$$

The proof still works if (C1) is replaced by (C1*). Even if one may think that being a triangular differs in positivity from being trilateral due to a hyperintensional distinction, it seems less plausible that the property of being a triangular circle differs in positivity from the property of being both a triangular circle and not a triangular circle. Since both properties involve the same concepts, it's less plausible that a hyperintensional distinction would make a difference for positivity.

¹⁶ Gustafsson (2020, p. 234) instead uses the axiom that the property of being self-different is not positive. But it seems that the reason for believing that the property of being self-different isn't positive is that it is contradictory. So (C2) seems more fundamental.

I have previously shown that these principles are sufficient to derive the first half of Gödel's proof — that is, that, if a property is positive, then it's possible that there exists something that has that property.¹⁷ Here, however, we will show that we can prove the necessary existence of something godlike without assuming any implausible axiological principles.

We define the property of being godlike as follows:¹⁸

$$(C3) \quad G(x) =_{\text{df}} \forall\phi(P(\phi) \supset \Box\phi(x)).$$

That is, something is godlike if and only if it has all positive properties necessarily. We adopt the following substantial axiological axiom:¹⁹

$$(C4) \quad P(G).$$

That is, the property of being godlike is positive. And we seek to prove that it's necessary that there exists something godlike:

$$(C5) \quad \Box\exists x G(x).$$

We can prove the following theorem:

THEOREM 1: Given axioms and definitions (C1), (C2), (C3), and (C4), we can derive (C5) in second-order system KB.

For proof, see Appendix A.

Our two general axiological principles, (C1) and (C2), both mirror standard principles of the logic of value.

Axiom (C1) mirrors the principle that logically equivalent states of affairs have the same intrinsic value.²⁰ There is a straightforward rationale

¹⁷ Gustafsson 2020, pp. 234–6.

¹⁸ Anderson (1990, pp. 294–5) has a biconditional instead of a conditional in definiens. Hájek (2002, p. 156) defines being godlike if and only if its necessary properties are those needed to have all positive properties. My definition is weaker in the sense that anything that is godlike by their definitions would also be godlike by mine. Note that only the left-to-right direction of my definition is used in the proofs. So we could replace definition (C3) by the following axiom:

$$(C3^*) \quad G(x) \supset (\forall\phi(P(\phi) \supset \Box\phi(x))).$$

Since (C3*) can be derived from both Anderson's and Hájek's definitions, we could also use their definitions. Anderson's definition, however, has the drawback that — unless tautological properties are positive — godlike beings are impossible.

¹⁹ Scott 1987, p. 257.

²⁰ Rescher 1966, p. 58 and Åqvist 1968, p. 259.

for the principle: If two properties mutually entail each other, any goodness and badness that is entailed by one of them is also entailed by the other. Hence the properties necessarily have the same advantages and disadvantages and so should be alike in positivity.²¹

Axiom (C2) mirrors the principle that contradictory states of affairs are not intrinsically good.²² And there is a rationale for the principle too: contradictions entail everything; so, for every good or bad thing that they entail, they also entail the complement. Contradictions are symmetrical in their relation to the good and the bad. Hence they are neither intrinsically good nor positive.²³

The proof relies on second-order system KB.²⁴ System K is normal modal logic, that is, propositional logic combined with the necessitation rule and the distribution axiom:²⁵

$$K \quad \Box(p \supset q) \supset (\Box p \supset \Box q).$$

System KB is system K combined with the Brouwerian axiom:²⁶

$$B \quad p \supset \Box \Diamond p.$$

Possibility, here, is defined as the dual of necessity:²⁷

$$\Diamond p =_{\text{df}} \neg \Box \neg p.$$

Note that system KB does not include the necessity axiom:²⁸

$$T \quad \Box p \supset p.$$

So one may wonder whether our assumptions allow us to derive that something godlike exists:

$$(C6) \quad \exists x G(x).$$

Nevertheless, we can prove the following corollary:

²¹ Gustafsson 2020, p. 235.

²² von Wright 1972, pp. 163–4 and Hansson 2001, p. 119.

²³ Gustafsson 2020, pp. 234–5.

²⁴ For system KB, see Chellas 1980, p. 131.

²⁵ For system K, see Chellas 1980, p. 131. For axiom K, see Feys 1950, p. 500 and Chellas 1980, p. 7.

²⁶ For axiom B, see Lewis and Langford 1932, p. 497 and Chellas 1980, p. 16.

²⁷ Aristotle *An. pr.* 1.13 32^a25; 2009, p. 18, Carnap 1947, p. 186, and Chellas 1980, p. 7.

²⁸ Carnap 1947, p. 186 and Chellas 1980, p. 6.

COROLLARY 1: Given axioms and definitions (C1), (C2), (C3), and (C4), we can derive (C6) in second-order system KB.

For proof, see Appendix B.

2. Varying domain

One worry about Theorem 1 is that it relies on a constant domain.²⁹ To get around this, we can move to a varying-domain setting with an existence predicate.³⁰ Let E be a predicate applied to individuals, with $E(x)$ read as ‘ x concretely exists’. Then we adopt the following definitions:³¹

$$\begin{aligned}\forall^E x \Phi &=_{\text{df}} \forall x (E(x) \supset \Phi). \\ \exists^E x \Phi &=_{\text{df}} \exists x (E(x) \& \Phi).\end{aligned}$$

We adopt the following axiological principles as axioms, where (V1) is a varying-domain analogue of (C1) and where (V2) is the same as (C2):³²

$$(V1) \quad \forall \phi \forall \psi \left(\Box \forall^E x (\phi(x) \equiv \psi(x)) \supset (P(\phi) \equiv P(\psi)) \right).$$

$$(V2) \quad \forall \phi \neg P(\phi \& \neg \phi).$$

We define the property of being godlike as before:³³

$$(V3) \quad G(x) =_{\text{df}} \forall \phi (P(\phi) \supset \Box \phi(x)).$$

²⁹ Another worry is that it seems to allow a parallel argument that there exists something devil-like. Replacing G and P with D and N respectively in (C1)–(C5) seems to yield a similarly plausible argument. If, however, concrete existence is positive and its negation is negative (as suggested in Smullyan 2002, p. 47), then the parallel negative variant of the varying-domain argument below is blocked.

³⁰ Anderson 1990, pp. 300–1n14.

³¹ Fitting and Mendelsohn 1998, p. 106 and Fitting 2002, p. 90.

³² Gustafsson 2020, p. 236n15. Like before, we could weaken (V1) as follows:

$$(V1^*) \quad \forall \phi \left(\Box \forall^E x (\phi(x) \equiv \phi \& \neg \phi(x)) \supset (P(\phi) \equiv P(\phi \& \neg \phi)) \right).$$

See note 15.

³³ And, like before, we only make use of the left-to-right direction of the definition — that is, we can replace definition (V3) with the following axiom:

$$(V3^*) \quad G(x) \supset \left(\forall \phi (P(\phi) \supset \Box \phi(x)) \right).$$

See note 18.

We adopt the following substantial axiological axiom:³⁴

(V4) $P(G \ \& \ E)$.

That is, the property of being godlike and concretely existing is positive. And we seek to prove that it's necessary that there concretely exists something godlike:

(V5) $\Box \exists^E x G(x)$.

We can prove the following theorem:

THEOREM 2: Given axioms and definitions (V1), (V2), (V3), and (V4), we can derive (V5) in second-order system KB.

For proof, see Appendix C.

Our assumptions also allow us to derive that something godlike exists:

(V6) $\exists^E x G(x)$.

That is, we can prove the following corollary:

COROLLARY 2: Given axioms and definitions (V1), (V2), (V3), and (V4), we can derive (V6) in second-order system KB.

For proof, see Appendix D.

3. Are these proofs compelling?

Given that (C1), (C2), (V1), and (V2) are plausible axiological principles, are these proofs compelling as arguments for their conclusion? Since (C3) and (V3) are definitions and the required modal system (second-order system KB) is fairly weak, it seems that whether we should accept the argument rests on whether we should accept the remaining axioms, (C4) and (V4). Unlike the other axiological axioms, (C4) and (V4) are not general formal axiological principles — that is, general principles about the formal structure of the positivity or value of properties. They are substantial axiological claims — that is, claims that a specific property is positive.

³⁴ Hájek 2002, p. 160.

If we grant that there are positive properties and that they are jointly consistent, it's plausible that G or $G \& E$ is positive. The trouble is that it remains unclear that we should grant this.³⁵

Moreover, (C4) and (V4) entail, given (C3) or (V3), that the property of necessarily having each positive property is positive. Given that we understand positive properties as “purely good”-making properties, G seems to entail more than just basic good-making properties unless necessarily having a positive property is itself positive.³⁶ It's unclear why having a possible property in other possible worlds would be good-making in this world.³⁷

Having noted these problems, the upshot of this paper is still that, *if* we grant that G or $G \& E$ is positive, the conclusion follows. And, unlike earlier Gödelian ontological proofs, we have shown this without relying on implausible axiological principles.

Appendices

A. Proof of Theorem 1

THEOREM 1: Given axioms and definitions

$$(C1) \quad \forall\phi\forall\psi\left(\Box\forall x(\phi(x) \equiv \psi(x)) \supset (P(\phi) \equiv P(\psi))\right),$$

$$(C2) \quad \forall\phi\neg P(\phi \& \neg\phi),$$

$$(C3) \quad G(x) =_{\text{df}} \forall\phi(P(\phi) \supset \Box\phi(x)), \text{ and}$$

$$(C4) \quad P(G),$$

we can derive, in second-order system KB,

³⁵ Anderson and Gettings 1996, p. 171, van Inwagen 2007, p. 144, and Gustafsson 2020, p. 238. Apart from the well known worries about the compatibility of traditional divine properties such as omniscience, omnipotence, and omnibenevolence, there is also the worry that some “purely good”-making properties may only be possessed by objects of different kinds. For example, the property of being a pleasure cannot be possessed by people whereas the property of being benevolent can only be possessed by people. To get around this problem, we can restrict P to properties that apply to a specific kind of object. (I thank Wlodek Rabinowicz for this point.)

³⁶ Gödel (1995b, p. 435), however, grants that ‘the necessity of a positive property is positive’.

³⁷ I thank Wlodek Rabinowicz for this point.

(C5) $\Box\exists x G(x)$.

*Proof.*³⁸ Assume, for proof by contradiction,

(6) $\neg\forall\phi(P(\phi) \supset \Diamond\exists x \phi(x))$. [Assumption]

From (6), we have

(7) $\exists\phi \neg(P(\phi) \supset \Diamond\exists x \phi(x))$. [(6)]

From (7), we have, by existential instantiation,

(8) $\neg(P(\phi') \supset \Diamond\exists x \phi'(x))$. [(6)]

From (8), we have

(9) $P(\phi')$ [(6)]

and

(10) $\neg\Diamond\exists x \phi'(x)$. [(6)]

From (10), we have, by the definition of possibility,

(11) $\Box\neg\exists x \phi'(x)$. [(6)]

From (11), we have, by necessitation and the K axiom,

(12) $\Box\forall x \neg\phi'(x)$. [(6)]

From (12), we have, by necessitation and the K axiom,

(13) $\Box\forall x(\phi'(x) \supset \phi' \ \& \ \neg\phi'(x))$. [(6)]

We have, by necessitation,

(14) $\Box\forall x(\phi' \ \& \ \neg\phi'(x) \supset \phi'(x))$. []

From (13) and (14), we have, by necessitation and the K axiom,

(15) $\Box\forall x(\phi'(x) \equiv \phi' \ \& \ \neg\phi'(x))$. [(6)]

³⁸ The proof up to (19) follows Gustafsson 2020, pp. 235–6.

From (C1) and (15), we have

$$(16) \quad P(\phi') \equiv P(\phi' \ \& \ \neg\phi'). \quad [(C1), (6)]$$

From (C2) and (16), we have

$$(17) \quad \neg P(\phi'). \quad [(C1), (C2), (6)]$$

From (9) and (17), we have

$$(18) \quad P(\phi') \ \& \ \neg P(\phi'). \quad [(C1), (C2), (6)]$$

Having derived a contradiction from (6), we can conclude, not depending on (6),

$$(19) \quad \forall\phi(P(\phi) \supset \Diamond\exists x \phi(x)). \quad [(C1), (C2)]$$

From (C4) and (19), we have

$$(20) \quad \Diamond\exists x G(x). \quad [(C1), (C2)]$$

Assume, for conditional proof,

$$(21) \quad G(a). \quad [\text{Assumption}]$$

From (C3) and (21), we have

$$(22) \quad \forall\phi(P(\phi) \supset \Box\phi(a)). \quad [(C3), (21)]$$

From (22), we have

$$(23) \quad P(G) \supset \Box G(a). \quad [(C3), (21)]$$

From (C4) and (23), we have

$$(24) \quad \Box G(a). \quad [(C3), (C4), (21)]$$

From (24), we have, not depending on (21),

$$(25) \quad G(a) \supset \Box G(a). \quad [(C3), (C4)]$$

From (25), we have

$$(26) \quad \forall x(G(x) \supset \Box G(x)). \quad [(C3), (C4)]$$

From (26), we have,

$$(27) \quad \exists x G(x) \supset \exists x \Box G(x). \quad [(C_3), (C_4)]$$

We have, by necessitation and the K axiom,

$$(28) \quad \exists x \Box G(x) \supset \Box \exists x G(x). \quad []$$

From (27) and (28), we have

$$(29) \quad \exists x G(x) \supset \Box \exists x G(x). \quad [(C_3), (C_4)]$$

From (29), we have,

$$(30) \quad \neg \Box \exists x G(x) \supset \neg \exists x G(x). \quad [(C_3), (C_4)]$$

From (30), we have, by necessitation,

$$(31) \quad \Box(\neg \Box \exists x G(x) \supset \neg \exists x G(x)). \quad [(C_3), (C_4)]$$

Assume, for proof by contradiction,

$$(32) \quad \neg \Box \exists x G(x). \quad [\text{Assumption}]$$

From (30) and (32), we have

$$(33) \quad \neg \exists x G(x). \quad [(C_3), (C_4), (32)]$$

From (33), we have, by the B axiom,

$$(34) \quad \Box \Diamond \neg \exists x G(x). \quad [(C_3), (C_4), (32)]$$

From (34), we have, by the definition of possibility,

$$(35) \quad \Box \neg \Box \neg \neg \exists x G(x). \quad [(C_3), (C_4), (32)]$$

From (35), we have, by necessitation and the K axiom,

$$(36) \quad \Box \neg \Box \exists x G(x). \quad [(C_3), (C_4), (32)]$$

From (31) and (36), we have, by the K axiom,

$$(37) \quad \Box \neg \exists x G(x). \quad [(C_3), (C_4), (32)]$$

From (20), we have, by the definition of possibility,

$$(38) \quad \neg \Box \neg \exists x G(x). \quad [(C1), (C2)]$$

From (37) and (38), we have

$$(39) \quad \Box \neg \exists x G(x) \ \& \ \neg \Box \neg \exists x G(x). \quad [(C1), (C2), (C3), (C4), (32)]$$

Having derived a contradiction from (32), we can conclude, not depending on (32),

$$(C5) \quad \Box \exists x G(x). \quad [(C1), (C2), (C3), (C4)]$$

■

B. Proof of Corollary 1

COROLLARY 1: Given axioms and definitions (C1), (C2), (C3), and (C4), we can derive, in second-order system KB,

$$(C6) \quad \exists x G(x).$$

Proof. Assume, for proof by contradiction,

$$(40) \quad \neg \exists x G(x). \quad [\text{Assumption}]$$

From (40), we have, by the B axiom,

$$(41) \quad \Box \Diamond \neg \exists x G(x). \quad [(40)]$$

From (41), we have, by the definition of possibility,

$$(42) \quad \Box \neg \Box \neg \neg \exists x G(x). \quad [(40)]$$

From (42), we have, by necessitation and the K axiom,

$$(43) \quad \Box (\Box \exists x G(x) \supset \neg \exists x G(x)). \quad [(40)]$$

From (C5), we have, by necessitation,

$$(44) \quad \Box \Box \exists x G(x). \quad [(C1), (C2), (C3), (C4)]$$

From (43) and (44), we have, by the K axiom,

$$(45) \quad \Box \neg \exists x G(x). \quad [(C1), (C2), (C3), (C4), (40)]$$

From (38) and (45), we have

$$(46) \quad \Box \neg \exists x G(x) \ \& \ \neg \Box \neg \exists x G(x). \quad [(C1), (C2), (C3), (C4), (40)]$$

Having derived a contradiction from (40), we can conclude, not depending on (40),

$$(C6) \quad \exists x G(x). \quad [(C1), (C2), (C3), (C4)]$$

■

C. Proof of Theorem 2

THEOREM 2: Given axioms and definitions

$$(V1) \quad \forall \phi \forall \psi \left(\Box \forall^E x (\phi(x) \equiv \psi(x)) \supset (P(\phi) \equiv P(\psi)) \right),$$

$$(V2) \quad \forall \phi \neg P(\phi \ \& \ \neg \phi),$$

$$(V3) \quad G(x) =_{\text{df}} \forall \phi (P(\phi) \supset \Box \phi(x)), \text{ and}$$

$$(V4) \quad P(G \ \& \ E),$$

we can derive, in second-order system KB,

$$(V5) \quad \Box \exists^E x G(x).$$

Proof. Assume, for proof by contradiction,

$$(47) \quad \neg \forall \phi (P(\phi) \supset \Diamond \exists^E x \phi(x)). \quad [\text{Assumption}]$$

From (47), we have

$$(48) \quad \exists \phi \neg (P(\phi) \supset \Diamond \exists^E x \phi(x)). \quad [(47)]$$

From (48), we have, by existential instantiation,

$$(49) \quad \neg (P(\phi') \supset \Diamond \exists^E x \phi'(x)). \quad [(47)]$$

From (49), we have

$$(50) \quad P(\phi') \quad [(47)]$$

and

$$(51) \quad \neg \diamond \exists^E x \phi'(x). \quad [(47)]$$

From (51), we have, by the definition of possibility,

$$(52) \quad \Box \neg \exists^E x \phi'(x). \quad [(47)]$$

From (52), we have, by necessitation and the K axiom,

$$(53) \quad \Box \forall^E x \neg \phi'(x). \quad [(47)]$$

From (53), we have, by necessitation and the K axiom,

$$(54) \quad \Box \forall^E x (\phi'(x) \supset \phi' \ \& \ \neg \phi'(x)). \quad [(47)]$$

We have, by necessitation,

$$(55) \quad \Box \forall^E x (\phi' \ \& \ \neg \phi'(x) \supset \phi'(x)). \quad []$$

From (54) and (55), we have, by necessitation and the K axiom,

$$(56) \quad \Box \forall^E x (\phi'(x) \equiv \phi' \ \& \ \neg \phi'(x)). \quad [(47)]$$

From (V1) and (56), we have

$$(57) \quad P(\phi') \equiv P(\phi' \ \& \ \neg \phi'). \quad [(V1), (47)]$$

From (V2) and (57), we have

$$(58) \quad \neg P(\phi'). \quad [(V1), (V2), (47)]$$

From (50) and (58), we have

$$(59) \quad P(\phi') \ \& \ \neg P(\phi'). \quad [(V1), (V2), (47)]$$

Having derived a contradiction from (47), we can conclude, not depending on (47),

$$(60) \quad \forall \phi (P(\phi) \supset \diamond \exists^E x \phi(x)). \quad [(V1), (V2)]$$

From (V4) and (60), we have

$$(61) \quad \Diamond \exists^E x G(x). \quad [(V1), (V2)]$$

Assume, for conditional proof,

$$(62) \quad G(a). \quad [\text{Assumption}]$$

From (V3) and (62), we have

$$(63) \quad \forall \phi (P(\phi) \supset \Box \phi(a)). \quad [(V3), (62)]$$

From (63), we have

$$(64) \quad P(G) \supset \Box (G \& E)(a). \quad [(V3), (62)]$$

From (V4) and (64), we have

$$(65) \quad \Box (G \& E)(a). \quad [(V3), (V4), (62)]$$

From (65), we have, not depending on (62),

$$(66) \quad G(a) \supset \Box (G \& E)(a). \quad [(V3), (V4)]$$

From (66), we have

$$(67) \quad \forall^E x (G(x) \supset \Box (G \& E)(x)). \quad [(V3), (V4)]$$

From (67), we have, by necessitation and the K axiom,

$$(68) \quad \exists^E x G(x) \supset \Box \exists^E x G(x). \quad [(V3), (V4)]$$

From (68), we have

$$(69) \quad \neg \Box \exists^E x G(x) \supset \neg \exists^E x G(x). \quad [(V3), (V4)]$$

From (69), we have, by necessitation,

$$(70) \quad \Box (\neg \Box \exists^E x G(x) \supset \neg \exists^E x G(x)). \quad [(V3), (V4)]$$

Assume, for proof by contradiction,

$$(71) \quad \neg \Box \exists^E x G(x). \quad [\text{Assumption}]$$

From (69) and (71), we have

$$(72) \quad \neg\exists^E x G(x). \quad [(V3), (V4), (71)]$$

From (72), we have, by the B axiom,

$$(73) \quad \Box\Diamond\neg\exists^E x G(x). \quad [(V3), (V4), (71)]$$

From (73), we have, by the definition of possibility,

$$(74) \quad \Box\neg\Box\neg\neg\exists^E x G(x). \quad [(V3), (V4), (71)]$$

From (74), we have, by necessitation and the K axiom,

$$(75) \quad \Box\neg\Box\exists^E x G(x). \quad [(V3), (V4), (71)]$$

From (70) and (75), we have, by the K axiom,

$$(76) \quad \Box\neg\exists^E x G(x). \quad [(V3), (V4), (71)]$$

From (61), we have, by the definition of possibility,

$$(77) \quad \neg\Box\neg\exists^E x G(x). \quad [(\neg V1), (\neg V2)]$$

From (76) and (77), we have

$$(78) \quad \Box\neg\exists^E x G(x) \ \& \ \neg\Box\neg\exists^E x G(x). \quad [(\neg V1), (\neg V2), (V3), (V4), (71)]$$

Having derived a contradiction from (71), we can conclude, not depending on (71),

$$(V5) \quad \Box\exists^E x G(x). \quad [(V1), (V2), (V3), (V4)]$$

■

D. Proof of Corollary 2

COROLLARY 2: Given axioms and definitions (V1), (V2), (V3), and (V4), we can derive, in second-order system KB.

$$(V6) \quad \exists^E x G(x).$$

Proof. Assume, for proof by contradiction,

$$(79) \quad \neg\exists^E x G(x). \quad [\text{Assumption}]$$

From (79), we have, by the B axiom,

$$(80) \quad \Box \Diamond \neg \exists^E x G(x). \quad [(79)]$$

From (80), we have, by the definition of possibility,

$$(81) \quad \Box \neg \Box \neg \neg \exists^E x G(x). \quad [(79)]$$

From (81), we have, by necessitation and the K axiom,

$$(82) \quad \Box (\Box \exists^E x G(x) \supset \neg \exists^E x G(x)). \quad [(79)]$$

From (V5), we have, by necessitation,

$$(83) \quad \Box \Box \exists^E x G(x). \quad [(V1), (V2), (V3), (V4)]$$

From (82) and (83), we have, by the K axiom,

$$(84) \quad \Box \neg \exists^E x G(x). \quad [(V1), (V2), (V3), (V4), (79)]$$

From (77) and (84), we have

$$(85) \quad \Box \neg \exists^E x G(x) \ \& \ \neg \Box \neg \exists^E x G(x). \quad [(V1), (V2), (V3), (V4), (79)]$$

Having derived a contradiction from (79), we can conclude, not depending on (79),

$$(V6) \quad \exists^E x G(x). \quad [(V1), (V2), (V3), (V4)]$$

■

I wish to thank Krister Bykvist and Wlodek Rabinowicz for valuable comments.

References

- Anderson, C. Anthony (1990) 'Some Emendations of Gödel's Ontological Proof', *Faith and Philosophy* 7 (3): 291–303.
- (2015) 'Gödel's 'Proof' for the Existence of God', in Snezana Lawrence and Mark McCartney, eds., *Mathematicians and Their Gods: Interactions Between Mathematics and Religious Beliefs*, pp. 279–289, Oxford: Oxford University Press.
- Anderson, C. Anthony and Michael Gettings (1996) 'Gödel's Ontological Proof Revisited', in Petr Hájek, ed., *Gödel '96: Logical Foundations of Mathematics, Computer Science and Physics—Kurt Gödel's Legacy*, pp. 167–172, Berlin: Springer.
- Åqvist, Lennart (1968) 'Chisholm-Sosa Logics of Intrinsic Betterness and Value', *Noûs* 2 (3): 253–270.
- Aristotle (2009) *Prior Analytics Book I*, ed. Gisela Striker, Oxford: Clarendon Press.
- Benzmüller, Christoph (2020) 'A (Simplified) Supreme Being Necessarily Exists, says the Computer: Computationally Explored Variants of Gödel's Ontological Argument', in Diego Calvanese, Esra Erdem, and Michael Thielscher, eds., *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning, KR 2020*, pp. 779–789, Somerset, NJ: IJCAI.
- (2022) 'Symbolic AI and Gödel's Ontological Argument', *Zygon* 57 (4): 953–962.
- Björdal, Frode (1999) 'Understanding Gödel's Ontological Argument', in Timothy Childers, ed., *The Logica Yearbook 1998*, pp. 214–217, Prague: Philosophia.
- Carnap, Rudolf (1947) *Meaning and Necessity: A Study in Semantics and Modal Logic*, Chicago: University of Chicago Press.
- Chellas, Brian F. (1980) *Modal Logic: An Introduction*, Cambridge: Cambridge University Press.
- Chisholm, Roderick M. and Ernest Sosa (1966) 'On the Logic of "Intrinsically Better"', *American Philosophical Quarterly* 3 (3): 244–249.
- Cook, Roy T. (2004) 'God, the Devil, and Gödel's Other Proof', in Libor Behounek, ed., *The Logica Yearbook 2003*, pp. 97–109, Prague: Philosophia.
- Cresswell, M. J. (1975) 'Hyperintensional Logic', *Studia Logica* 34 (1): 25–38.
- Feys, R. (1950) 'Les systèmes formalisés des modalités aristotéliennes',

- Revue philosophique de Louvain* 48 (1): 478–509.
- Fitting, Melvin (2002) *Types, Tableaus, and Gödel's God*, Dordrecht: Kluwer.
- Fitting, Melvin and Richard L. Mendelsohn (1998) *First-Order Modal Logic*, Dordrecht: Kluwer.
- Gödel, Kurt (1987) 'Ontologischer Bewies', in Judith Jarvis Thomson, ed., *On Being and Saying: Essays for Richard Cartwright*, pp. 256–257, Cambridge, MA: MIT Press.
- (1995a) 'Ontological Proof', in Solomon Feferman, John W. Jr. Dawson, Warren Goldfarb, Charles Parsons, and Robert N. Solovay, eds., *Kurt Gödel Collected Works Volume III: Unpublished Essays and Lectures*, pp. 403–404, Oxford: Oxford University Press.
- (1995b) 'Texts Relating to the Ontological Proof', in Solomon Feferman, John W. Jr. Dawson, Warren Goldfarb, Charles Parsons, and Robert N. Solovay, eds., *Kurt Gödel Collected Works Volume III: Unpublished Essays and Lectures*, pp. 429–437, Oxford: Oxford University Press.
- Gustafsson, Johan E. (2020) 'A Patch to the Possibility Part of Gödel's Ontological Proof', *Analysis* 80 (2): 229–240.
- Hájek, Petr (2002) 'A New Small Emendation of Gödel's Ontological Proof', *Studia Logica* 71 (2): 149–164.
- Hansson, Sven Ove (2001) *The Structure of Values and Norms*, Cambridge: Cambridge University Press.
- Hartshorne, Charles (1962) *The Logic of Perfection and Other Essays in Neoclassical Metaphysics*, La Salle, IL: Open Court.
- Johnson, David (1999) *Hume, Holism, and Miracles*, Ithaca, NY: Cornell University Press.
- (2004) *Truth without Paradox*, Lanham, MD: Rowman & Littlefield.
- Kovač, Srećko (2003) 'Some Weakened Gödelian Ontological Systems', *Journal of Philosophical Logic* 32 (6): 565–588.
- Lewis, Clarence I. and Cooper H. Langford (1932) *Symbolic Logic*, New York: Century Company.
- Magari, Roberto (1988) 'Logica e Teofilia: Osservazioni su una dimostrazione attribuita a Kurt Gödel', *Notizie di Logica* 7 (4): 11–20.
- Maydole, Robert (2003) 'The Modal Perfection Argument for the Existence of a Supreme Being', *Philo* 6 (2): 299–313.
- Plantinga, Alvin (1974a) *God, Freedom, and Evil*, New York: Harper & Row.
- (1974b) *The Nature of Necessity*, Oxford: Clarendon Press.

- Pruss, Alexander R. (2009) 'A Gödelian Ontological Argument Improved', *Religious Studies* 45 (3): 347–353.
- (2012) 'A Gödelian Ontological Argument Improved Even More', in Mirosław Szatkowski, ed., *Ontological Proofs Today*, pp. 203–211, Frankfurt: Ontos Verlag.
- Rescher, Nicholas (1966) 'Semantic Foundations for the Logic of Preference', in Nicholas Rescher, ed., *The Logic of Decision and Action*, pp. 37–62, Pittsburgh: University of Pittsburgh Press.
- Scott, Dana (1987) 'Gödel's Ontological Proof', in Judith Jarvis Thomson, ed., *On Being and Saying: Essays for Richard Cartwright*, pp. 257–258, Cambridge, MA: MIT Press.
- Smullyan, Raymond (2002) *Some Interesting Memories: A Paradoxical Life*, Davenport, IA: Thinkers' Press.
- Sobel, Jordan Howard (2004) *Logic and Theism: Arguments for and against Beliefs in God*, Cambridge: Cambridge University Press.
- (2006a) 'On Gödel's Ontological Proof', in Henrik Lagerlund, Sten Lindström, and Rysiek Sliwinski, eds., *Modality Matters: Twenty-Five Essays in Honour of Krister Segerberg*, pp. 397–421, Uppsala: Uppsala Universitet.
- (2006b) 'To My Critics with Appreciation: Responses to Taliaferro, Swinburne, and Koons', *Philosophia Christi* 8 (2): 249–292.
- van Inwagen, Peter (2007) 'Some Remarks on the Modal Ontological Argument', in Matthias Lutz-Bachmann and Thomas M. Schmidt, eds., *Metaphysik heute: Probleme und Perspektiven der Ontologie / Metaphysics Today: Problems and Prospects of Ontology*, pp. 132–145, Freiburg: Alber.
- von Wright, Georg Henrik (1972) 'The Logic of Preference Reconsidered', *Theory and Decision* 3 (2): 140–169.