A Strengthening of the Consequence Argument for Incompatibilism*

Johan E. Gustafsson†

abstract. The aim of the Consequence Argument is to show that, if determinism is true, no one has, or ever had, any choice about anything. In the stock version of the argument, its two premisses state that (i) no one is, or ever was, able to act so that the past would have been different and (ii) no one is, or ever was, able to act so that the laws of nature would have been different. This stock version fails, however, because it requires an invalid inference rule. The standard response is to strengthen both premisses by replacing ‘would’ with ‘might’. While this response ensures validity, it weakens the argument, since it strengthens the premisses. I show that we can do better: We can keep the weak reading of one premiss and just strengthen the other. This provides two versions of the Consequence Argument which are stronger than the standard revision.

The aim of the Consequence Argument is to show that, if determinism is true, no one has, or ever had, any choice about anything. In a bare-bones version, the argument runs as follows:

If determinism is true, the remote past and the laws of nature jointly entail each one of our acts. Neither the remote past nor the laws of nature are up to us. Therefore, if determinism is true, our acts aren’t up to us.

The standard modal version, by Peter van Inwagen, is stated in terms of an unavoidability operator (van Inwagen 1983: 93–95):

\[ Np =_{df} p, \text{ and no one has, or ever had, any choice about whether } p. \]


† I would be grateful for any thoughts or comments on this paper, which can be sent to me at johan.eric.gustafsson@gmail.com.
This version of the argument relies on two inference rules. The first concerns the relation between unavoidability and logical necessity, with logical necessity represented by ‘□’:

\[
\text{Alpha: From } \Box p, \text{ deduce } Np. 
\]

The second rule concerns the transfer of unavoidability, with ‘⊃’ representing material implication:

\[
\text{Beta: From } N(p \supset q) \text{ and } Np, \text{ deduce } Nq. 
\]

Let \(P_0\) be a true proposition expressing a total state of the world at a time before anyone had any choice about anything. Let \(L\) be the conjunction of the laws of nature. And let \(P\) be an arbitrary true proposition. Then the argument can be stated as follows:

\textit{The Consequence Argument (standard modal version)}

\begin{align*}
(1) & \quad \Box((P_0 \& L) \supset P) \quad \text{A consequence of determinism} \\
(2) & \quad \Box(P_0 \supset (L \supset P)) \quad \text{From (1) by normal modal logic} \\
(3) & \quad N(P_0 \supset (L \supset P)) \quad \text{From (2) by Alpha} \\
(4) & \quad NP_0 \quad \text{Premiss, fixity of the past} \\
(5) & \quad N(L \supset P) \quad \text{From (3) and (4) by Beta} \\
(6) & \quad NL \quad \text{Premiss, fixity of the laws} \\
(7) & \quad NP \quad \text{From (5) and (6) by Beta}
\end{align*}

The validity of this argument, however, depends on how, in the definition of \(Np\), we understand someone’s having a choice about whether \(p\). We need to distinguish the following readings (McKay and Johnson 1996: 119 and Carlson 2000: 280):

\[
N_W p =_{df} p, \text{ and no one is, or ever was, able to act so that } p \text{ would be false.} \\
N_M p =_{df} p, \text{ and no one is, or ever was, able to act so that } p \text{ might be false.}
\]

That a person \(S\) is able to act at \(t\) so that \(p\) would (might) be false means that \(S\) is able to act at \(t\) in such a way that, if \(S\) were to act in that way at \(t\), then \(p\) would (might) be false (Carlson 2002: 393).

The premisses of the Consequence Argument are notably weaker given the \(N_W\) reading than given the \(N_M\) reading.\(^1\) But, given the \(N_W\) reading,

\(^1\) Given the stronger \(N_M\) reading of the premisses, we do, however, get the stronger conclusion \(N_M P\), rather than just \(N_W P\).
there are counter-examples to Beta (Widerker 1987: 38–39, McKay and Johnson 1996: 115–16, and Carlson 2003). Suppose, for instance, that

\[ S \] does not toss a coin, although she can do so. Let \( p = \) “the coin does not land heads”, and let \( q = \) “the coin is not tossed”. \( S \) is not able to ensure that \( p \) is false. She cannot ensure that the coin lands heads; if she were to toss it, it might land heads, but it might just as well land tails. Hence, \( \neg p \) is true. Similarly, \( S \) cannot ensure the falsity of \( p \supset q \). If she were to toss the coin, \( p \supset q \) might be false (since the coin might land tails), but it might equally well be true (since the coin might land heads). Thus, \( \neg(p \supset q) \) is also true. However, \( \neg q \) is false, since \( S \) can ensure the falsity of \( q \) by tossing the coin.\(^2\) (Carlson 2003: 732)

\(^2\) To avoid this counter-example, O’Connor (1993: 209) suggests the following revision:

**Beta’**: From \( \neg(p \supset q) \), \( \neg p \), and \( p \) is made true earlier than \( q \), deduce \( \neg q \).

Nonetheless, Carlson puts forward a more complicated counter-example that also applies to Beta’:

At time \( t_0 \), \( S \) can either press or not press a certain button, which is connected to a coin-tossing mechanism. If she presses the button, the mechanism will toss the coin twice, first at \( t_1 \), and then again at a later time, \( t_2 \). If \( S \) does not press the button, the coin will only be tossed once, at \( t_1 \). Suppose that \( S \) presses the button, and that the coin lands heads in both tosses. Let \( p = \) “the coin is tossed at \( t_1 \), and lands heads”, and let \( q = \) “the coin is tossed at \( t_2 \), and lands heads”. Since the coin might land tails in the \( t_1 \)-toss, whatever \( S \) does at \( t_0 \), \( \neg p \) is true. […] If \( S \) would refrain from pressing the button, the coin might land tails in the \( t_1 \)-toss, in which case \( p \supset q \) would still be true. Hence, \( \neg(p \supset q) \) is true. On the other hand, \( \neg q \) is false. By not pressing the button, \( S \) can ensure the falsity of \( q \). Moreover, \( p \) is made true earlier than \( q \). (2000: 284)

A similar revision, however, avoids that counter-example too:

**Beta”**: From \( \neg(p \supset q) \), \( \neg p \), and no one has any choice before \( p \) was already made true about whether \( q \), deduce \( \neg q \).

Given that \( P_0 \) is made true before anyone had any choice about anything, one could rely on Beta” in a version of the Consequence Argument. Yet the plausibility of Beta” depends on the plausibility that, once a proposition has been made true, one is unable to act so that it might be false. To see this, suppose we change Carlson’s example so that \( S \)’s choice whether to press the button occurs not at \( t_0 \) but at \( t_{1.5} \) (that is, after \( t_1 \) but before \( t_2 \)) and the coin still did land heads in the \( t_1 \)-toss but, if \( S \) were to refrain from pressing at \( t_{1.5} \), the coin might have landed tails in the \( t_1 \)-toss. If this change were plausible, we would have a counter-example to Beta”. So, to plausibly rely on Beta” in the Consequence Argument, we need the strong fixity-of-the-past premiss – that is,
Given the $N_M$ reading, Beta is not open to these counter-examples (Carlson 2000: 286–87). So, to rid the Consequence Argument from counter-examples to its validity, it appears that we must give its premisses the stronger $N_M$ reading. This is also van Inwagen’s move. I shall show, however, that we can do better: We only need to give one of the premisses the stronger $N_M$ reading. The other premiss can have the weaker $N_W$ reading.

Consider the following inference rules for $N_W$ and $N_M$:  

\[
\begin{align*}
\text{Alpha}_M: & \quad \text{From } \Box p, \text{ deduce } N_M p. \\
\text{Beta}_{MW}: & \quad \text{From } N_M (p \supset q) \text{ and } N_W p, \text{ deduce } N_W q. \\
\text{Beta}_{WM}: & \quad \text{From } N_W (p \supset q) \text{ and } N_M p, \text{ deduce } N_W q.
\end{align*}
\]

\[
\text{Alpha}_M \text{ is just Alpha given the } N_M \text{ reading. It needs, I think, no further argument – if } p \text{ is a logically necessary truth, it seems clear that one cannot act so that } p \text{ might be false. But one might worry that either Beta}_{MW} \text{ or Beta}_{WM} \text{ is open to some undiscovered counter-example, as Beta was given the } N_W \text{ reading.} \]

With these rules, however, we have more reason to be optimistic. There are two cogent arguments for their validity.

For the validity of Beta$_{MW}$, consider the following argument, which takes the form of a proof by contradiction:

\[
\begin{align*}
(1) & \quad N_M (p \supset q) \quad \text{Premiss} \\
(2) & \quad N_W p \quad \text{Premiss} \\
(3) & \quad \sim N_W q \quad \text{Assumption} \\
(4) & \quad q \quad \text{From (1) and (2)} \\
(5) & \quad \text{Someone is or was able to act so that } q \text{ would be false.} \quad \text{From (3) and (4)} \\
(6) & \quad \text{For all persons } S, \text{ all possible acts } \phi, \text{ and all present and past times } t \text{ such that } S \text{ is able to } \phi \text{ at } t, \text{ if } S \text{ were to } \phi \text{ at } t, \text{ then } S \text{ would act at } t \text{ so that } p \supset q \text{ would be true.} \quad \text{From (1)}
\end{align*}
\]

$N_M P_0$ – to block this kind of counter-example. And then Beta” has no advantage over Beta$_{WM}$ (defined later).


4 In McKay and Johnson 1996: 119, Beta$_{MW}$ is called 2 and Beta$_{WM}$ is called 1.

5 Although they haven’t found any counter-examples, Finch and Warfield (1998: 525) question whether these rules are valid. Van Inwagen (2004: 223–24; 2017b: 12–13) expresses similar worries.
(7) Someone is or was able to act so that \( \sim q \& (p \supset q) \) would be true.

(8) Someone is or was able to act so that \( p \) would be false.

(9) \( \sim N_W p \) From (8)

(10) \( N_W p \& \sim N_W p \) From (2) and (9)

(11) \( N_W q \) From (3) and (10)

We have that \( N_M (p \supset q) \) and \( N_W p \) together entail \( N_W q \). Hence Beta_{MW} is valid. Still, some of the inferences in this argument might need some explanation. For the inference from (1) and (2) to (4), note that \( N_M (p \supset q) \) entails \( p \supset q \) and that \( N_W p \) entails \( p \). The idea behind the inference from (1) to (6) is simply that, if one is unable to do anything such that \( p \supset q \) might be false, then it must be that, for the things one is able to do, \( p \supset q \) would be true if one were to do any of those things.\(^6\)\(^7\) The inference from (5) and (6) to (7) is perhaps controversial. It follows from (5) that some person \( S \) is able to do something at some time \( t \) such that \( \sim q \) would be true if \( S \) were to do that thing at \( t \), and it follows from (6) that, if \( S \) were to do that thing at \( t \), then \( S \) would act at \( t \) so that \( p \supset q \) would be true. Hence \( S \) is able to act at \( t \) so that both \( \sim q \) and \( p \supset q \) would be true.\(^8\) And then \( S \) is able to act at \( t \) so that \( \sim q \& (p \supset q) \) would be true.

\(^6\) This inference is valid given the right-to-left direction of Lewis's (1973: 21) duality definition of 'might' counterfactuals:

Duality: \( p \diamond q =_{df} \sim (p \Box \sim q) \),

that is,

Duality Right-to-Left: From \( \sim (p \Box \sim q) \), deduce \( p \diamond q \).

We don't need the more controversial left-to-right direction, that is,

Duality Left-to-Right: From \( p \diamond q \), deduce \( \sim (p \Box \sim q) \).

Given Duality Left-to-Right, Stalnaker's (1981: 100) seemingly non-paradoxical claim that John might not have come to the party if he had been invited, but I believe he would have come amounts to a Moorean paradox.\(^7\)

\(^7\) The restriction to present and past times in (6) is needed unless we strengthen the N operators so that \( Np \) also rules out that anyone will have a choice about whether \( p \). That strengthening wouldn't make the premisses any less plausible.

\(^8\) This inference is valid if the following inference rule is valid:

Conjunction Composition: From \( r \Box p \) and \( r \Box q \), deduce \( r \Box p \& q \).

This rule is valid in the systems of Stalnaker (1968) and Lewis (1973); see Chellas 1975: 138, 150115 for some further examples.
There is an analogous argument that Beta\textsubscript{WM} is valid:

\begin{align*}
(1) & \quad N_W(p \supset q) & \text{Premiss} \\
(2) & \quad N_M p & \text{Premiss} \\
(3) & \quad \neg N_W q & \text{Assumption} \\
(4) & \quad q & \text{From (1) and (2)} \\
(5) & \quad \text{Someone is or was able to act so that } q \text{ would be false.} & \text{From (3) and (4)} \\
(6) & \quad \text{For all persons } S, \text{ all possible acts } \phi, \text{ and all present and past times } t \text{ such that } S \text{ is able to } \\
& \quad \phi \text{ at } t, \text{ if } S \text{ were to } \phi \text{ at } t, \text{ then } S \text{ would act at } t \text{ so that } p \text{ would be true.} & \text{From (2)} \\
(7) & \quad \text{Someone is or was able to act so that } p \& \neg q \text{ would be true.} & \text{From (5) and (6)} \\
(8) & \quad \text{Someone is or was able to act so that } p \supset q \text{ would be false.} & \text{From (7)} \\
(9) & \quad \neg N_W(p \supset q) & \text{From (8)} \\
(10) & \quad N_W(p \supset q) \& \neg N_W(p \supset q) & \text{From (1) and (9)} \\
(11) & \quad N_W q & \text{From (3) and (10)} \\
\end{align*}

In much the same way as in the argument for the validity of Beta\textsubscript{MW}, we have that $N_W(p \supset q)$ and $N_M p$ together entail $N_W q$. Hence Beta\textsubscript{WM} is valid.

With Alpha\textsubscript{M}, Beta\textsubscript{MW}, and Beta\textsubscript{WM}, we can give either one of the two fixity premisses the weaker $N_W$ reading and adopt one of the following versions of the Consequence Argument:\textsuperscript{9}

\textsuperscript{9} One might object that, in these versions of the Consequence Argument, we could replace Alpha\textsubscript{M} and Beta\textsubscript{MW} by the following variant of Beta suggested by Widerker (1987: 41):

Beta\textsubscript{GW}: From $\Box(p \supset q)$ and $N_W p$, deduce $N_W q$.

This would let us infer (5) directly from (2) and (4) and thus skip lemma (3). Yet the plausibility of Beta\textsubscript{GW} depends on the plausibility of logically necessary propositions' being unavoidable in the strong sense, that is, it depends on the plausibility of Alpha\textsubscript{M}. It's clearer to treat the transfer of unavoidability separately from the relation between unavoidability and logical necessity, since these are two distinct, controversial issues. See also footnote 24.
The Consequence Argument (weak-fixity-of-the-past version)

(1) \( \Box((P_0 \& L) \supset P) \)  
    A consequence of determinism

(2) \( \Box(P_0 \supset (L \supset P)) \)  
    From (1) by normal modal logic

(3) \( N_M(L \supset (P_0 \supset P)) \)  
    From (2) by \( \text{Alpha}_M \)

(4) \( N_WP_0 \)  
    Premiss, weak fixity of the past

(5) \( N_W(L \supset P) \)  
    From (3) and (4) by \( \text{Beta}_{MW} \)

(6) \( N_M L \)  
    Premiss, strong fixity of the laws

(7) \( N_WP \)  
    From (5) and (6) by \( \text{Beta}_{WM} \)

The Consequence Argument (weak-fixity-of-the-laws version)

(1) \( \Box((P_0 \& L) \supset P) \)  
    A consequence of determinism

(2) \( \Box(L \supset (P_0 \supset P)) \)  
    From (1) by normal modal logic

(3) \( N_M(L \supset (P_0 \supset P)) \)  
    From (2) by \( \text{Alpha}_M \)

(4) \( N_WL \)  
    Premiss, weak fixity of the laws

(5) \( N_W(P_0 \supset P) \)  
    From (3) and (4) by \( \text{Beta}_{MW} \)

(6) \( N_M P_0 \)  
    Premiss, strong fixity of the past

(7) \( N_WP \)  
    From (5) and (6) by \( \text{Beta}_{WM} \)

Thus we have two new options for raising the price of compatibilism. Of these, the weak-fixity-of-the-laws version is, I think, especially compelling.

Given a necessitarian view of laws of nature, both the weak and the strong fixity-of-the-laws premisses seem plausible. But consider a Humean view of laws, where these laws hold contingently and merely describe (rather than prescribe) how the world develops. Given this kind of view, there seems to be little reason to accept the strong fixity-of-the-laws premiss, that is, \( N_M L \) (Vihvelin 1988: 231; 1990: 376–77 and Beebee and Mele 2002: 206–10). To see this, suppose, as seems fairly plausible, that, if it’s still contingent shortly after \( p \) is made true whether \( \sim q \) will be made true, then the ‘might’ counterfactual ‘If it were the case that \( p \), it might be the case that \( \sim q \)’ is true, even if \( p \) and \( q \) are true.\(^{10}\) (If, for example, \( S \) will toss a coin at \( t \) which will in fact land heads, it still seems plausible that, if \( S \) were to toss the coin \( t \), it might land tails.) Then, given a Humean view, there’s a trivial argument against the strong

\(^{10}\) This claim is ruled out by Lewis’s (1973: 21, 26) account of counterfactuals, which combines the duality definition of ‘might’ counterfactuals (see footnote 6) with

**Conjunction Conditionalization:** From \( p \& q \), deduce \( p \mathcal{R} q \).

But the claim is compatible with Lewis’s (1986: 63–65) would-be-possible account of ‘might’ counterfactuals.
fixity-of-the-laws premiss. Plausibly, you are able to do what you actually
do. And, if you are able to do what you actually do and shortly afterwards
it’s still contingent what laws will turn out to be the actual laws of nature,
then you’re able to act so that $L$ might be false. Hence, on a Humean
view of laws, $N_M L$ could plausibly be denied.\footnote{Compare Finch and Warfield’s (1998: 526) analogous point about the Mind Argument.}

But denying the weak fixity-of-the-laws premiss – that is, $N_W L$ – is
less trivial. I shall highlight two problems. To see them, assume first that
$N_W L$ is false. If $N_W L$ is false, someone must at some point have been
able to act otherwise than they actually did. Let $t_1$ be the first time $t$ such
that, for some true proposition $P$, someone was able to act at $t$ so that
$P$ would be false.\footnote{Or, if time is dense, let $t_1$ be an arbitrarily early such time.} And let $P_0$ be a true proposition expressing the total
state of the world at a time before, but arbitrarily close to, $t_1$. From $N_M P_0$,
we have that the total state of the world before $t_1$ is fixed.\footnote{Lewis (1981: 116–17) only takes the remote past to be fixed. But, as argued by Ginet (1990: 107–10) and Huemer (2000: 541–44), the reason the remote past seems fixed is that it is the past and not because it is, in addition, remote. Lewis (1979: 462–63, 468) argues fairly convincingly that, if one were to act otherwise, there would have been a divergence miracle shortly before the act. But it doesn’t follow that one is able to act so that there would have been a divergence miracle shortly before the act; see Ekstrom 2000: 50 and Huemer 2000: 541–42.} To block an
application of the weak-fixity-of-the-laws version of the Consequence Ar-
gument at $t_1$, we have that $N_W L$ is false at $t_1$ and hence that some person $S$
is able to act at $t_1$ so that $L$ would be false.

The first problem is a deterministic variant of the luck problem for
libertarianism.\footnote{Beebee and Mele (2002: 220–21) spell out the parallels between the luck problem for libertarianism and this variant for Humean compatibilism.} Since $S$ does not in fact act at $t_1$ so that $L$ would be false, no feature of the total state of the world before $t_1$ – including $S$’s character,
deliberation, and mental states – ensures that $S$ acts at $t_1$ so that $L$ would
be false. But then, if $S$ had acted at $t_1$ so that $L$ would have been false, it is
hard to see what could explain the difference in acting, since everything
about $S$ and the state of the world before $t_1$ would have been exactly the
same as it was in the actual world. Given a Humean view of the laws of
nature, no difference in these laws could explain the difference in acting,
since the laws then depend on what happens, rather than the other way
around.\footnote{One might object that the laws could explain the difference retroactively. Note, however, that a key part of what gives laws their explanatory power is their overall simplicity.} Hence it seems that the acting so that $L$ would be false would
have been due not to S's agency but to chance. Without chance turning out otherwise than it actually did, there's nothing S could have done to render L false at t₁. It seems that S is merely able to act at t₁ so that L might be false – acting so that L would be false at t₁ would have required chance to turn out otherwise – hence it seems that S is unable to act at t₁ so that L would be false.¹⁶

The second problem is that S's ability to act at t₁ so that L would be false requires that S has the incredible ability to break the laws of nature (Beebee and Mele 2002: 212–13). To see this, note that, since P₀ expresses the total state of the world at a time just before t₁, we have from NₘP₀ that the actual state of the world just before t₁ is exactly the same as it would have been just before t₁ if S had acted at t₁ so that L would have been false.¹⁷ Hence the laws wouldn't have been broken by some miracle before t₁; the laws would be broken by S's act.¹⁸ Thus rejecting NₘL at t₁ requires that S has the incredible ability to do something at t₁ such that, if S did it, S's act would break what are in fact the laws of nature.¹⁹,²⁰

While these two problems for denying NₘL are perhaps surmount-

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¹⁶ Note that I'm not assuming here that acting so that p would be false requires that one has control over whether p, that is, by also being able to instead act so that p would be true. Thus I'm not ruling out that one is able to act so that p would be false if so acting requires chance to turn out a certain way. If chance turns out the required way, one might arguably be able to act so that p would be false. My claim is merely that, if acting so that p would be false requires chance to turn out a certain way and chance doesn't turn out in the required way, then one is unable to act so that p would be false.

¹⁷ See footnote 13.

¹⁸ Thus we close Lewis's (1981: 119) loophole that the act that would render P₀ & L false 'would not itself falsify any law – not if all the requisite lawbreaking were over and done with beforehand'; see Huemer 2000: 541–44; 2004.

¹⁹ That is, S would have rendered the law false in Lewis's (1981: 120) strong sense.

²⁰ One might wonder why – if one accepts a Humean view of laws – one wouldn't be willing to just bite this bullet and accept, as a consequence of the contingency of the laws, that we have this ability. Isn't this just a consequence of the overall Humean set-up? No. The contingency of the laws doesn't require that they are up to us; that's a further tenet of Humean compatibilism.
able, they are not problems for denying $N_M L$ given a Humean view.\footnote{Berofsky (2012: 252–54) offers some potential solutions these problems.} On a Humean view, it’s contingent what laws will turn out to be the actual laws of nature. So, as we argued earlier, no matter how you were to act, you would be acting so that $L$ might be false. And, since this holds even if you don’t act otherwise than you actually do, there is no need to explain any difference in acting compared to how you actually act; so the first problem doesn’t apply. Regarding the second problem, note that acting so that $L$ might be false doesn’t require any incredible ability if, regardless of what you do, $L$ might be false. Thus, on a Humean view, denying $N_M L$ raises neither of these problems, while denying $N_W L$ raises both.

The weak-fixity-of-the-past version has perhaps also an advantage. There are two main lines of argument in support of the fixity-of-the-past premiss. One is based on the idea that the past is fixed in the sense that, when we evaluate counterfactuals, we try to keep the (remote) past fixed.\footnote{See, for example, Lewis 1979: 461–72, Ginet 1990: 107–10, and Huemer 2000: 541–44. Lewis (1979: 458, 463) allows the immediate past to not be fixed; see footnote 13.} This counterfactual-fixity argument seems to support not just the weak version of the premiss but also the strong version. The other line of argument is based on the idea that the past isn’t up to us, that is, the idea that the past is not under our present control.\footnote{See, for example, Mele 1995: 249.} This present-control argument supports $N_W P_0$ but not $N_M P_0$. It doesn’t support $N_M P_0$, because the past might fail to be fixed in a way that is independent of our acts. That is, regardless of what anyone did, many possible pasts including the actual past might be the past – so $P_0$ might be true, and $P_0$ might be false. And then the past wouldn’t be up to us: no one would be able to act so that $P_0$ would be false. $N_W P_0$ would be true, but $N_M P_0$ would be false. Thus one of the main lines of argument in support of the fixity of the past only supports the weak version of the premiss.

Denying the weak fixity premisses is hence more challenging than denying their strong variants. While this doesn’t amount to a defence of incompatibilism, it shows that the new strengthened versions of the Consequence Argument are more problematic for compatibilism than the version that requires both strong fixity of the past and strong fixity of the laws.\footnote{Carlson (2000: 284–85) provides a cogent argument for Beta_{GW} (defined in footnote 9). From Beta_{GW} and Beta_{WM}, one can derive the following agglomeration rule:}
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References


Agglomeration\textsubscript{MW}: From $N_M p$ and $N_W q$, deduce $N_W (p \& q)$.

Proof: From $\Box (q \supset (p \supset (p \& q)))$ and $N_W q$, we have, by Beta\textsubscript{GW}, $N_W (p \supset (p \& q))$; then, from $N_W (p \supset (p \& q))$ and $N_M p$, we have, by Beta\textsubscript{WM}, $N_W (p \& q)$. With Agglomeration\textsubscript{MW} and Beta\textsubscript{GW}, one could argue in one of the following ways:

*The Consequence Argument (agglomerative weak-fixity-of-the-laws version)*

1. $\Box ((P_0 \& L) \supset P)$ A consequence of determinism
2. $N_M P_0$ Premiss, strong fixity of the past
3. $N_W L$ Premiss, weak fixity of the laws
4. $N_W (P_0 \& L)$ From (2) and (3) by Agglomeration\textsubscript{MW}
5. $N_W P$ From (1) and (4) by Beta\textsubscript{GW}

*The Consequence Argument (agglomerative weak-fixity-of-the-past version)*

1. $\Box ((L \& P_0) \supset P)$ A consequence of determinism
2. $N_M L$ Premiss, strong fixity of the laws
3. $N_W P_0$ Premiss, weak fixity of the past
4. $N_W (L \& P_0)$ From (2) and (3) by Agglomeration\textsubscript{MW}
5. $N_W P$ From (1) and (4) by Beta\textsubscript{GW}

These agglomerative versions of the Consequence Argument seem on a par with the versions suggested earlier; one can derive Beta\textsubscript{MW} and Beta\textsubscript{WM} from Agglomeration\textsubscript{MW} and Beta\textsubscript{GW}. Proofs: From $N_M (p \supset q)$ and $N_W p$, we have, by Agglomeration\textsubscript{MW}, $N_W ((p \supset q) \& p)$; then, from $\Box (((p \supset q) \& p) \supset q)$ and $N_W ((p \supset q) \& p)$, we have, by Beta\textsubscript{GW}, $N_W q$. And, from $N_M p$ and $N_W (p \supset q)$, we have, by Agglomeration\textsubscript{MW}, $N_W (p \& (p \supset q))$; then, from $\Box ((p \& (p \supset q)) \supset q)$ and $N_W (p \& (p \supset q))$, we have, by Beta\textsubscript{GW}, $N_W q$. One might object that the agglomerative approach has an advantage since it doesn’t rely on Alpha\textsubscript{M}. But, as I argue in footnote 9, relying on Beta\textsubscript{GW} rather than Alpha\textsubscript{M} and Beta\textsubscript{MW} – is a drawback rather than an advantage. (I thank Erik Carlson for suggesting this agglomerative approach.)


