Does the Collapsing Principle Rule Out Borderline Cases?＊

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ABSTRACT. If '*F*' is a predicate, then '*F*er than' or 'more *F* than' is a correspond-ing comparative relational predicate. Concerning such comparative relations, John Broome's Collapsing Principle states that, for any *x* and *y*, if it is false that *y* is *F*er than *x* and not false that *x* is *F*er than *y*, then it is true that *x* is *F*er than *y*. Luke Elson has recently put forward two alleged counter-examples to this principle, al-legedly showing that it yields contradictions if there are borderline cases. In this paper, I argue that the Collapsing Principle does not rule out borderline cases, but I also argue that it is implausible.

For two decades, John Broome has defended the Collapsing Principle as a principle of logic for comparative relations. Here, a *comparative relation* should be understood in a technical, linguistic sense. Broome explains:

> Take any monadic predicate such 'dangerous' or 'sunny in the morn-ing'. For generality, designate it with the schematic letter '*F*'. We can often form from *F* a dyadic predicate, or relation, designated by 'more *F* than'. For example, we form 'more dangerous than' and 'more sunny in the morning than'. Call this the 'comparative rela-tion' of *F*. In English, when '*F*' is a short adjective, 'more *F* than' generally has the synonym '*F*er than'. Irregularly, 'more good than' has the synonym 'better than'.[1]

[1] John Broome *Weighing Lives* (Oxford, 2004), p. 50. Broome writes as if there is a unique comparative relation of *F*. Yet there are two: one for superiority in *F*ness, formed by 'more *F* than'; and one for inferiority in *F*ness, formed by 'less *F* than'—see Rodney Huddleston and Geoffrey K. Pullum, *The Cambridge Grammar of the English Language* (Cambridge, 2002), p. 1100. While we shall focus on superiority comparatives, the dis-cussion will also apply to inferiority comparatives, changing what needs to be changed. Perhaps Broome meant that there's only one relation *R* here, designated by both 'more *F*' and 'less F' so that *R* holds between items *x* and *y* if and only if '*x* is more *F* than *y*' is true if and only if '*y* is less *F* than *x*' is true. (I thank Krister Bykvist for this suggestion.)

The Collapsing Principle concerns cases where it is indeterminate whether a comparative relation holds between some items—that is, cases where it's neither true nor false that the relation holds between the items. Broome states the principle as follows:

> *The Collapsing Principle*
> For any x and y, if it is false that y is Fer than x and not false that x is Fer than y, then it is true that x is Fer than y.[2]

Luke Elson has recently put forward two alleged counter-examples to the Collapsing Principle. Elson claims that, if there are borderline cases, the Collapsing Principle entails contradictions.

In this paper, I shall defend two claims. Against Elson, I shall argue that

(1)   The Collapsing Principle need not entail contradictions if there are borderline cases.

But, against Broome, I shall still argue that

(2)   The Collapsing Principle is somewhat implausible.

One might question the importance of (1) given (2). Yet, even if (2) is true, (1) matters for one of the central issues in ethical theory: The Collapsing Principle is the main premise of Broome's argument against value incomparability. Broome argues that value incomparability rules out vagueness in the betterness relation; and, since there is vagueness in the betterness relation, there is no value incomparability.[3] If, however, the Collapsing Principle rules out borderline cases by itself, it would assume the point at issue in Broome's argument, making it a non-starter. On the other hand,

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[2] This is Broome's 'special version' ('Is Incommensurability Vagueness?', ed. R. Chang, *Incommensurability, Incomparability, and Practical Reason* (Cambridge, MA, 1997), pp. 67–89, at 74); his 'general version' is stated in terms of degrees of truth ('Incommensurability', p. 77). Elsewhere, he states a somewhat different versions of the principle; see J. Broome, *Weighing Lives* (Oxford, 2004), p. 174 and 'Reply to Rabinowicz', *Philosophical Issues* 19 (2009), pp. 412–17, at 416. The differences between these versions, however, won't be crucial for our discussion. Broome, 'Incommensurability', pp. 74–75, also provides some arguments for the Collapsing Principle; see, however, Johan E. Gustafsson, 'Indeterminacy and the Small-Improvement Argument', *Utilitas* 25 (2013), pp. 433–445, at 437–438, for some objections to these arguments.

[3] Broome, 'Incommensurability', pp. 73–74.

given (1) and (2), Broome's argument might still have some limited cogency. In addition, even if the Collapsing Principle and Broome's argument are both invalid, we should try to find out why.

1. 'Settaller Than'

Although Elson presents his alleged counter-examples to the Collapsing Principle as two versions of the same basic example, these examples differ somewhat in their structure. Hence we shall discuss them separately. For the first example, Elson asks us to consider a comparative predicate, defined as follows:

(3)   Set X is settaller than set $Y =_{df}$ set X contains more tall men than set Y.[4]

Given this new predicate, compare the following sets:

A, which contains ten tall men and nothing else,

B, which contains ten tall men, one borderline tall man (call him 'Tallish'), and nothing else, and

C, which contains eleven tall men and nothing else.

Since B contains at least as many men as A, it is false that A is settaller than B. And, since it's indeterminate whether B contains ten or eleven tall men, it's not false that B contains more tall men than A and thus not false that B is settaller than A. Then, according to Elson, the Collapsing Principle yields that B is settaller than A. And, if it's true that B is settaller than A, it follows by (3) that it's true that B contains more tall men than A. So it must be true that Tallish is a tall man.[5]

We then apply the same kind of reasoning to the comparison of B and C. Since C contains at least as many tall men as B, it's false that B is settaller than C. And, since it's indeterminate whether B contains ten or eleven men, it's not false that C contains more tall men than B and thus

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[4] Luke Elson, 'Borderline Cases and the Collapsing Principle', *Utilitas* 26 (2014), pp. 51–60, at 55.

[5] Elson, 'Borderline Cases', p. 55. This conclusion seems to conflict with the original assumption that Tallish is merely a borderline tall man. If we already have a contradiction, then the second half of the example is superfluous. It makes no difference for my objections whether we adopt this shorter version of the example or the longer one, because my objections apply to the first half of the example.

not false that *C* is settaller than *B*. Then, according to Elson, the Collapsing Principle yields that *C* is settaller than *B*. And, if it's true that *C* is settaller than *B*, it follows by (3) that it's true that *C* contains more tall men than *B*. So it must be false that Tallish is a tall man, which contradicts the earlier claim that it's true that Tallish is a tall man. The upshot of Elson's argument is that, if there are borderline cases like Tallish, the Collapsing Principle yields contradictions.[6]

Elson addresses the worry that 'settaller than' is too artificial to be a compelling counter-example to the Collapsing Principle. He writes

> Is the predicate 'is settaller than' objectionably artificial? This is not a promising line of objection. First, the predicate is not all *that* outré: there is nothing special about counting the number of tall men in various sets. Moreover, the collapsing principle is intended to be fully general, and not limited to natural-language plausible predicates.[7]

I shall argue, however, that the problem with 'settaller than' is not that it is an *artificial* comparative; the problem is that it is not a *comparative*.

Remember that comparative relations in Broome's sense are formed by modifying a monadic predicate *F* by a marker of comparative grade, such as 'more *F* than' or '*F*er than'. Elson's 'settaller than' ends in '-er than', but there doesn't seem to be any monadic predicate 'settall' of which 'settaller than' is a comparative. The logical form of 'contains more tall men than' does not match that of a comparative relation. First, it is of the form 'contains more *F* than' rather than 'more *F* than'.[8] Second, the *F* that is modified by 'more' is not a predicate but a plural noun phrase—'more'

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[6] Elson, 'Borderline Cases', p. 55.

[7] Elson, 'Borderline Cases', p. 56.

[8] Compare Moore's objection to Brentano's fitting-attitude analysis of 'good' and of 'better' in G. E. Moore, 'Review of Franz Brentano, *The Origin of the Knowledge of Right and Wrong*', *The International Journal of Ethics* 14 (1903), pp. 115–123, at 118:

> His first suggestion is that since *"good"* means "worthy to be loved," *"better"* must mean "worthy of *more love*".... It does not seem to have occurred to him that it must mean *"more worthy* of love," …

If 'better than' is a comparative (which it seems to be), then the same objection should also rule out contemporary versions of Brentano's approach, such as those by Joshua Gert, 'Value and Parity', *Ethics* 114 (2004), pp. 492–510, at 505, or Wlodek Rabinowicz, 'Value Relations', *Theoria* 74 (2008), pp. 18–49, at 38. They both define that '*x* is better than *y*' as 'it is rationally required that *x* is preferred to *y*'. Their definiendum is a comparative but their definiens lacks the structure of a comparative.

4

in 'contains more tall men than' modifies 'tall men' and not just 'tall'. So 'more' is used here in the sense 'a greater number of' rather than as a marker of comparative grade. It is a conflation between these two senses of 'more' that drives the example: When we learn that B is settaller than A, we learn by (3) that B contains a greater number of tall men than A, rather than that B is more settall than A (whatever that might mean).

Even though 'settaller than' is a dyadic relation that involves a comparison between its relata, it's not a comparative relation.[9] And, if so, it's not a counter-example to the Collapsing Principle, which is only put forward as a principle of logic for comparative relations. This is not an ad hoc restriction. Whether a dyadic relation is a comparative or just a relation that involves a comparison between its relata is relevant for what logical principles hold for that relation. Take, for example, Broome's claim that it is necessary that comparative relations are transitive.[10] Dyadic relations of the form 'a little bit more F than' involve comparisons between their relata and are clearly non-transitive. Comparatives, in the linguistic sense, of the form 'more F than' are more plausibly transitive, however.[11]

Perhaps Elson's 'settaller than' example could, with some changes, be turned into a proper counter-example to the Collapsing Principle. To attempt this kind of fix, we need to first define a monadic predicate 'settall' and then form a comparative of this predicate. The challenge is to find a definition of 'settall' which has a comparative that would support Elson's line of argument. One suggestion for 'settall' could be

(4)   Set X is settall $=_{df}$ set X contains many tall men.

[9] This point also applies to Chang's alleged counter-example based on the relation 'much heavier than', which is of the form 'much Fer than' rather than 'Fer than'. A comparative 'Fer than' holds if the first relata has a higher degree of Fness than the second relata; Ruth Chang, *Making Comparisons Count* (London, 2002), p. 166. The relation 'much Fer than', on the other hand, does not have this kind of structure; it holds when the first relata has a much higher degree of Fness than the second relata. In 'much Fer than', 'much' modifies the comparative 'Fer than'; it is not itself part of a comparative. To see this, note that comparatives in English can be modified by 'much', 'far', 'somewhat', 'slightly', and other modifiers; see Huddleston and Pullum, *The Cambridge Grammar*, p. 1131. For example, *Smith is somewhat heavier than Jones* is grammatical; but *Smith is somewhat much heavier than Jones* is not. Hence it should be clear that 'much heavier than' is not a comparative. So Chang's alleged counter-example doesn't work against the Collapsing Principle.

[10] Broome, *Weighing Lives*, p. 50.

[11] Broome, *Weighing Lives*, pp. 50–63, provides an extended defence of comparative relations' being necessarily transitive against several alleged counter-examples.

Perhaps 'contains many tall men' is not a predicate that allows for comparative grades, since 'more contains many tall men' is ungrammatical. One might suggest that a comparative relation corresponding to (4) could be

(5)   Set X is settaller than set Y =$_{df}$ set X contains to a greater extent many tall men than set Y.

Nevertheless, insofar as 'B contains to a greater extent of many tall men than A' can be given a clear meaning, its being true doesn't seem to logically entail its being true that B contains more tall men than A, since 'to a greater extent' modifies 'contains' rather than 'many'. The addition of the borderline tall Tallish in B to the tall men in A doesn't make it (i) clearly true that B contains more tall men than A, but it might arguably make it (ii) clearly true that B contains to a greater extent many tall men than A. Elson's argument needs (i), but only (ii) follows from the Collapsing Principle.

A better way to revise the example is to use the predicate 'populous' and the comparative 'more populous than'. We can then revise Elson's example as follows:[12]

> A is a country of ten million inhabitants.
>
> B is a country of ten million inhabitants and one borderline inhabitant living on the border (call her 'Borderline').
>
> C is a country of ten million and one inhabitants.

Since it seems false that A is more populous than B and not false that B is more populous than A, the Collapsing Principle entails that B is more populous than A. Like before, one might then argue that, since A has ten million inhabitants and B is more populous than A, B must have more than ten million inhabitants. So Borderline is an inhabitant of B. And, since it seems false that B is more populous than C and not false that C is more populous than B, the Collapsing Principle entails that C is more populous than B. Again, one might then argue that, since C is more populous than B and C has ten million and one inhabitants, B cannot have more than ten million inhabitants. Hence Borderline is not an inhabitant of B, and we have a contradiction.

This revised line of argument against the Collapsing Principle is more compelling, but much the same objections apply. This revised example, I

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[12] I thank Erik Carlson for suggesting this revision.

shall argue, only illustrates that the Collapsing Principle is implausible, not that it entails contradictions. It seems plausible that, if it's clearly true that *B* is more populous than *A*, it's also clearly true that *B* contains more inhabitants than *A*. While I agree that this claim is plausible, I think one could reject it without contradiction. One might claim that, even if it's clearly true that *B* is more populous than *A*, it still isn't clearly true that *B* contains more inhabitants than *A*. This could hold if populousness comes in finer degrees than the addition of one person, for example, the addition of one borderline person. The revised line of argument doesn't show that this idea is contradictory. If populousness comes in finer degrees in this manner, we can reject the inference from that *B* is more populous than *A* to that *B* contains more inhabitants than *A* and the inference from that *C* is more populous than *B* to that *C* contains more inhabitants than *B*. And, if these inferences are invalid, then this revised line of argument doesn't show that the Collapsing Principle yields contradictions if there are borderline cases.

2. Large Holiday Destinations

Let's turn to Elson's second alleged counter-example, which he claims is a version of the same general counter-example as the first.[13] As we shall see, however, the second example differs in structure from the first. Unlike the first example, the second example concerns a comparative relation, namely, 'better as a holiday destination'. Hence my objection to the 'settaller than' example does not apply.

Suppose that Elson prefers visiting large countries and, accordingly, that being a large country is a good-making feature of holiday destinations. And suppose that China, Ireland, and France are equally good as p. 489 holiday destinations in all relevant respects except size but that China is clearly large, Ireland is clearly not large, and France is borderline large. Elson's argument is divided into two rounds; the first concerns the comparison between Ireland and France.

> Round 1. It is false that Ireland is better than France (since 'Ireland is large and France is not' is false), but not false that France is better than Ireland (since 'France is large and Ireland is not' is borderline). By the collapsing principle, it is true that France is better

[13] Elson, 'Borderline Cases', pp. 56–57.

7

than Ireland. Given my preferences, it must be true that France is a large country. It could not have been borderline large after all.[14]

To reach the conclusion that France is a large country, Elson assumes that

(6)   If countries x and y are equally good as holiday destinations in all relevant respects except size and x is better as a holiday destination than y, then x is large and y is not large.

The second round concerns the comparison between France and China.

> Round 2. It is false that France is better than China, and not false that China is better than France. Therefore, it is true that China is better than France. Given my preferences, it must be false that France is a large country. It could not have been borderline large after all.
>
> Contradiction.[15]

Like in the first round, Elson assumes (6) to derive the second conjunct in the contradiction that it's true and also false that France is a large country. Having reached this contradiction, we are only forced to give up one of the assumptions. But, rather than giving up the Collapsing Principle, we could give up (6). Instead of (6), one might, for example, accept the following weaker claim:

(7)   If countries x and y are equally good as holiday destinations in all relevant respects except size and x is better as a holiday destination than y, then

- x is large and y is not large, or
- x is large and y is borderline large, or
- x is borderline large and y is not large.

If we accept (7) rather than (6), we can avoid the contradiction in the second example even if the Collapsing Principle holds, since we then block the conclusion that it's true that France is large and also the contrary conclusion that it's false.

     One might object that one could just stipulate that (6) holds as a part of the example's set-up. But then the above objection could instead be

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[14] Elson, 'Borderline Cases', p. 57.
[15] Elson, 'Borderline Cases', p. 57.

levelled against the plausibility of the example. The contradiction only follows from the Collapsing Principle given (6), so we can still maintain that the Collapsing Principle doesn't yield contradictions as long as we can deny the plausibility of (6).

Hence neither of Elson's alleged counter-examples to the Collapsing Principle shows that the principle yields contradictions if there are borderline cases.

3. The Balding Cavalier

If we removed the attempt to derive a contradiction from the Collapsing Principle, Elson's second example would be similar to a more straightforward counter-example.

> *The Balding Cavalier*
> Suppose that *A* and *B* are two prospective cavaliers, identical in every relevant aspect except that it's indeterminate whether *B* is bald but clear that *A* is not bald. And suppose that, for superficial reasons, baldness contributes negatively to one's goodness as a cavalier. Then, surely, *B* is not better than *A*. But, since it's indeterminate whether *B* is bald, it's indeterminate whether *B* differs from *A* in any relevant respect that contributes negatively to *B*'s goodness. Thus it should be indeterminate whether *A* is better than *B*.[16]

[16] Johan E. Gustafsson, *Preference and Choice* (PhD dissertation, Royal Institute of Technology, 2011), p. 26, and 'Indeterminacy', p. 436. Henrik Andersson, *How It All Relates: Exploring the Space of Value Comparisons* (PhD dissertation, Lund University, 2017), p. 92, points out the similarity between Elson's second example and the Balding Cavalier. The Balding Cavalier is a variation of the following kind of example by Erik Carlson, 'Broome's Argument against Value Incomparability', *Utilitas* 16 (2004), pp. 220–224, p. 224:

> suppose that *A* and *B* are two identical alarm clocks, except that *A* is waterproof, and *B* is not. Is *A* a better alarm clock than *B*? There may be no definite answer, since it may be indeterminate whether water resistance is a good-making characteristic of artefacts that are not very likely to come into contact with water. It is clear, however, that *B* is not better than *A*, since *A*'s being waterproof definitely does not detract from its goodness as an alarm clock.

Carlson's example, however, relies on its being indeterminate which feature are good making. Broome, 'Reply to Rabinowicz', p. 417, objects that it couldn't be indeterminate whether a certain feature contributes to the value of an item. This objection does not

Here, the charge against the Collapsing Principle isn't that it yields contradictions but merely that it does not fit with this seemingly plausible story.

Erik Carlson suggests that one might resist this counter-example if one relies on the following monadic variant of the Collapsing Principle:

> *The Monadic Collapsing Principle*
> For any x and y, if it is false that y is F and not false that x is F, then it is true that x is Fer than y.[17]

To be at all plausible, the Monadic Collapsing Principle should be restricted to gradable predicates F that predicate a plain degree of Fness.[18] The idea is that, if it's false that A is bald and not false that B is bald, then B must be balder than A, contradicting the above story.

Nevertheless, the Monadic Collapsing Principle seems to be open to similar counter-examples as the (dyadic) Collapsing Principle. Carlson offers the following variation of the Balding Cavalier:

> Let us slightly modify Gustafsson's cavalier case, and assume that B is definitely bald, whereas A is a borderline case of baldness. In all other relevant respects, the two cavaliers are identical. Suppose also that, given their other properties, not being bald is necessary and sufficient for A or B to qualify as a good cavalier. It is thus false that B is good, and indeterminate whether A is good. The monadic collapsing principle then implies that A is definitely better than B. But this seems false, since it is indeterminate whether A lacks the property, viz. baldness, whose absence would constitute the only relevant difference, as compared to B.[19]

This variation seems to rely on the same kind of intuition as the Balding Cavalier. Hence the Monadic Collapsing Principle conflicts with the same kind of counter-examples as the (dyadic) Collapsing Principle. It seems, therefore, point-assuming to rely on the Monadic Collapsing Principle in a defence of the (dyadic) Collapsing Principle from these counter-examples.

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apply to the Balding Cavalier.

[17] Erik Carlson, 'Vagueness, Incomparability, and the Collapsing Principle', *Ethical Theory and Moral Practice* 16 (2013), pp. 449–463, at 454.

[18] Here, I follow the terminology of Huddleston and Pullum, *The Cambridge Grammar*, p. 1162n39; plain degree has traditionally been called 'positive degree'.

[19] Carlson, 'Vagueness', pp. 454–455.

Henrik Andersson tries to defend the Monadic Collapsing Principle from this objection. He discusses a different yet analogous case, where

> Alf and Beth are identical in all relevant aspects except that […] Beth is narrow-minded and it is indeterminate whether Alf is narrow-minded. Because of this, for Alf and Beth, given the other properties they possess, not being narrow-minded is a necessary and sufficient condition to qualify as a good philosopher.[20]

Andersson objects that

> Carlson is mistaken in that the only relevant difference between Alf and Beth is that it is indeterminate whether Alf is narrow-minded while it is determinate that Beth is not. Since it is indeterminate whether Alf is narrow-minded it is also *not false that Alf is not narrow-minded*. And since it is *false that Beth is not narrow-minded* it must, in accordance with the monadic collapsing principle, be *true that Alf is more not narrow-minded than Beth*, or more naturally: *Beth is more narrow-minded than Alf*.[21]

Andersson's objection is, I think, unconvincing. First, it is point-assuming to defend the Monadic Collapsing Principle with the help of that principle, or to rely on the same kind of inference.[22] Second, *Alf's being more not narrow-minded than Beth* isn't equivalent to *Beth's being more narrow-minded than Alf*. In the former, 'more' modifies the predicate 'not narrow-minded', which seems to require that negations allow degrees. All we get is that Beth is *not narrow-minded* to a lesser degree than Alf. Without further assumptions, we cannot derive that Beth is more narrow-minded than Alf.

In conclusion, Elson's first example doesn't work, since—as defined—'set-taller than' is not a comparative. And the 'more populous' revision and Elson's second example need some further assumptions, which can be consistently rejected. Hence these examples do not show that the Collapsing Principle yields contradictions if there are borderline cases. The Balding Cavalier is a less ambitious counter-example—it only tries to show that

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[20] Henrik Andersson, 'Propping Up the Collapsing Principle', *Ethical Theory and Moral Practice* 18 (2014), pp. 475–486, at 482–483.
[21] Andersson, 'Propping Up', p. 483.
[22] I thank Erik Carlson for this point.

p. 492

11

the Collapsing Principle is implausible. But, as I have argued, this less ambitious example is cogent. The upshot is that, while the Collapsing Principle is implausible, it doesn't seem to rule out borderline cases.[23]